



# intro to infectious disease modeling

Vittoria Colizza

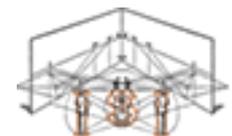
INSERM & UPMC  
Paris, France

ISI Foundation,  
Turin, Italy

**Inserm**

Institut national  
de la santé et de la recherche médicale

**UPMC**  
PARIS UNIVERSITAS



# infectious diseases

- ▶ some background
- ▶ principles, notation, classification
- ▶ modeling (I): homogeneous mixing
- ▶ modeling (II): contact networks
- ▶ modeling (III): space
- ▶ surveillance
- ▶ applications to H1N1 pdm

infectious diseases:  
some background

# mortality

**Influenza and pneumonia**  
1918: 588.5 deaths per 100,000

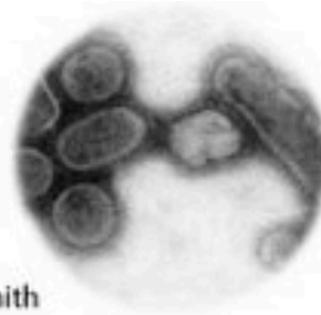
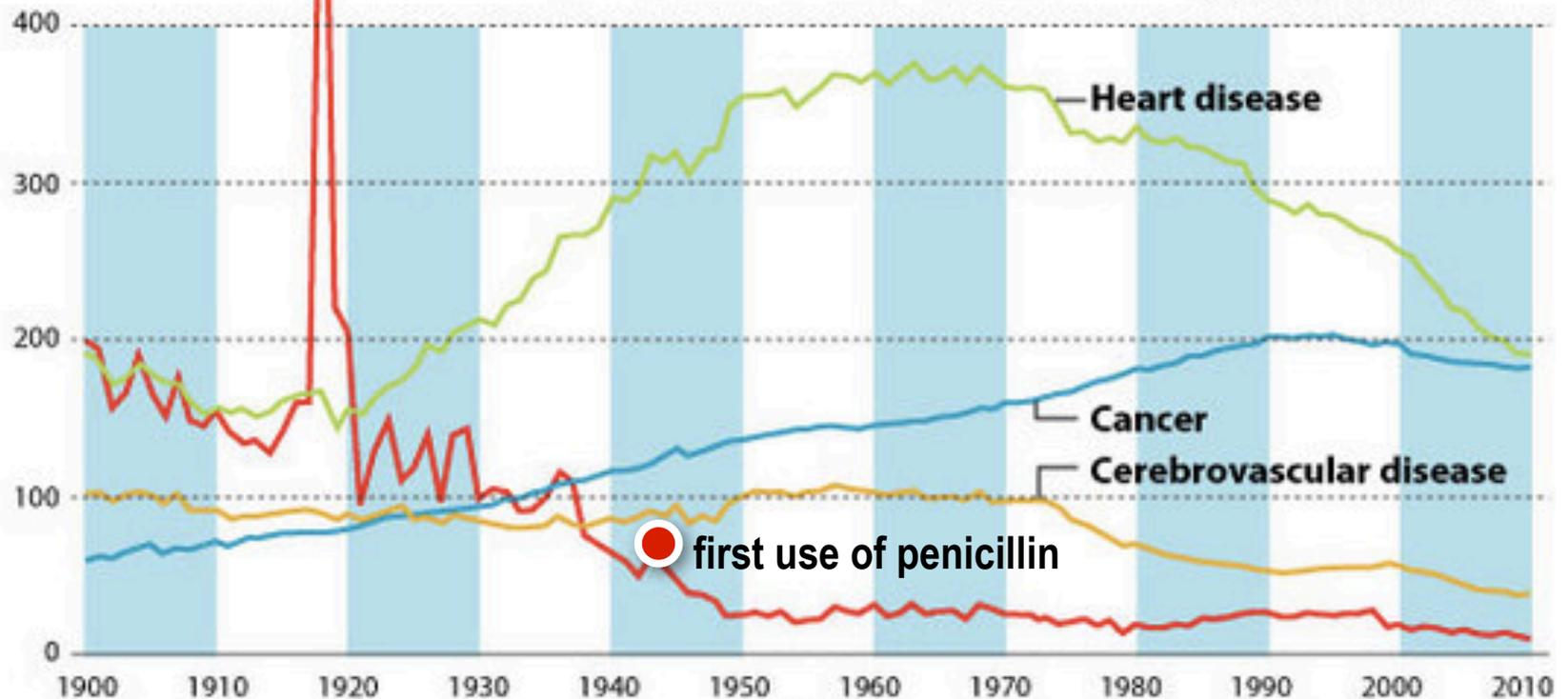


Photo Credit:  
CDC/ Cynthia Goldsmith

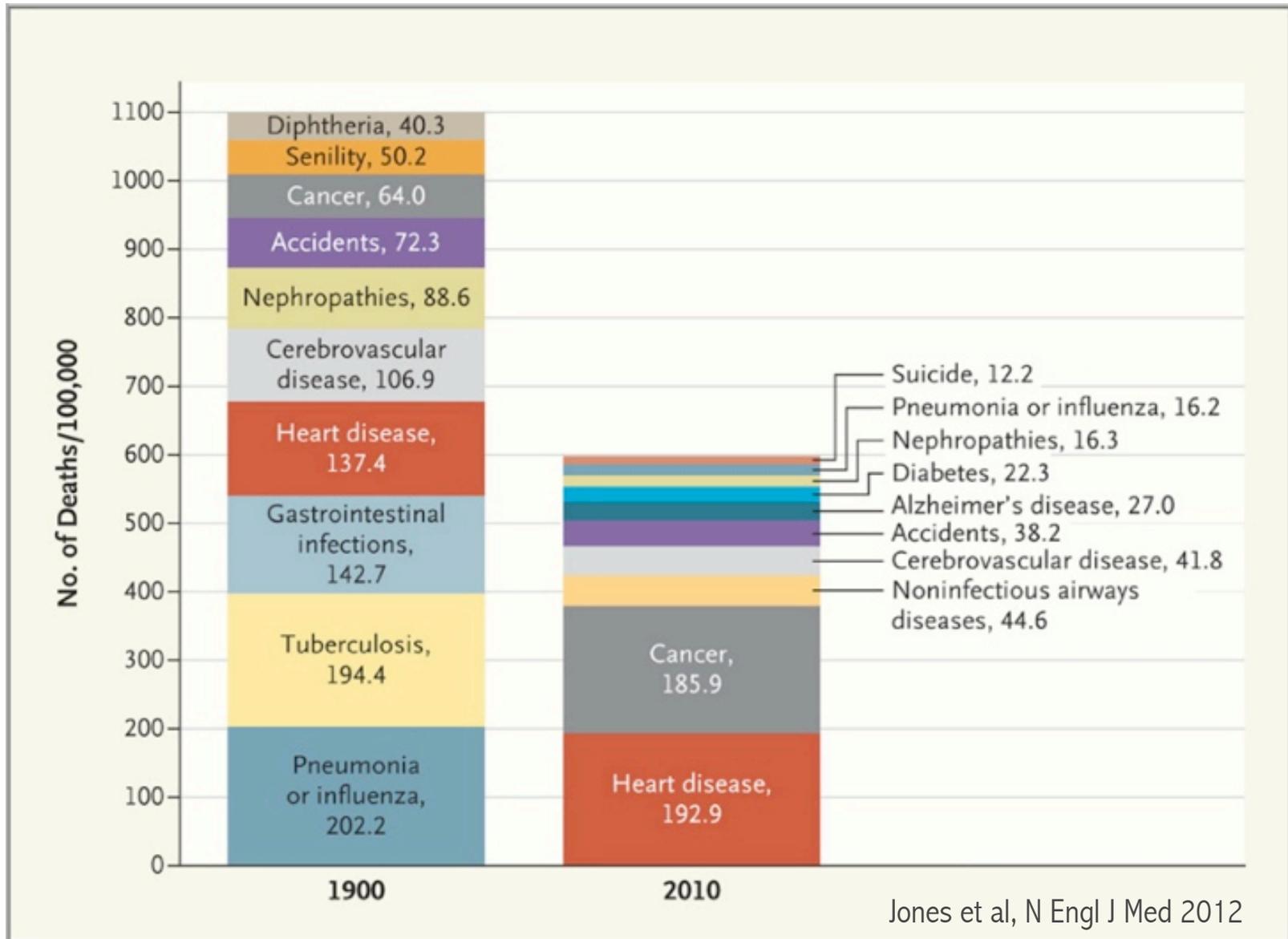
*Between 50 million and 130 million people died worldwide in the 1918 influenza pandemic, making it one of the deadliest natural disasters in history.*



SOURCES: CENTERS FOR DISEASE CONTROL AND PREVENTION,  
NEW ENGLAND JOURNAL OF MEDICINE

R. TORO / © LiveScience.com

# 10 leading causes of death (US)



# infectious disease impact worldwide

INFECTIOUS DISEASES *are the* **2<sup>ND</sup>**  
LEADING CAUSE *of* DEATH WORLDWIDE  
*after heart disease & are* RESPONSIBLE *for*  
MORE DEATHS ANNUALLY THAN CANCER<sup>2</sup>

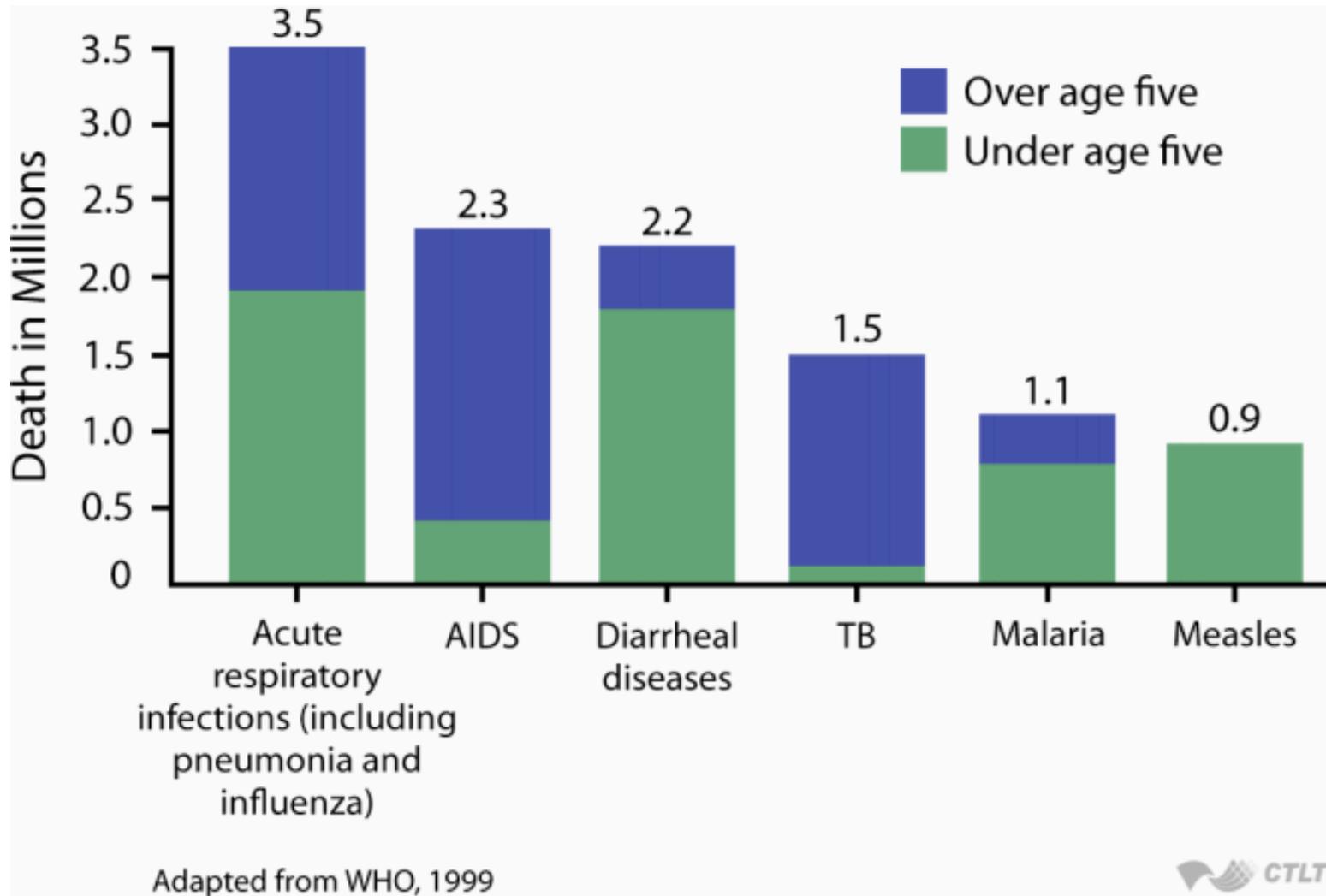
WHO, [www.bcm.edu/molvir/id](http://www.bcm.edu/molvir/id)

INFECTIOUS DISEASES CAUSE EACH YEAR<sup>2</sup>:  
**16.2%** *of* GLOBAL  
DEATHS 

**2/3** *of* ALL DEATHS AMONG  
CHILDREN YOUNGER  
THAN AGE **FIVE**<sup>3</sup> 

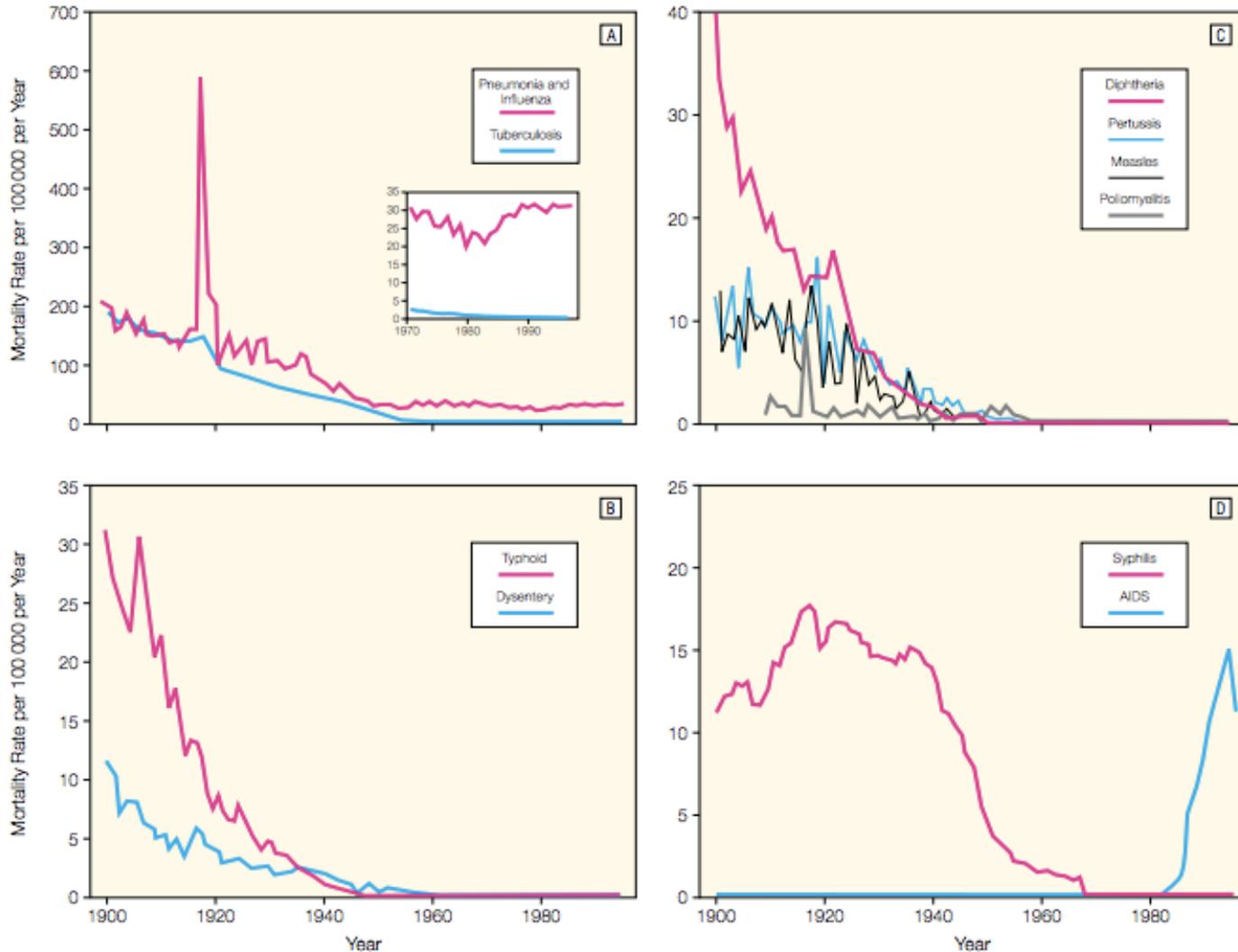
NIH, [www.nih.gov/about/discovery/infectious\\_diseases/](http://www.nih.gov/about/discovery/infectious_diseases/)

# infectious disease impact worldwide

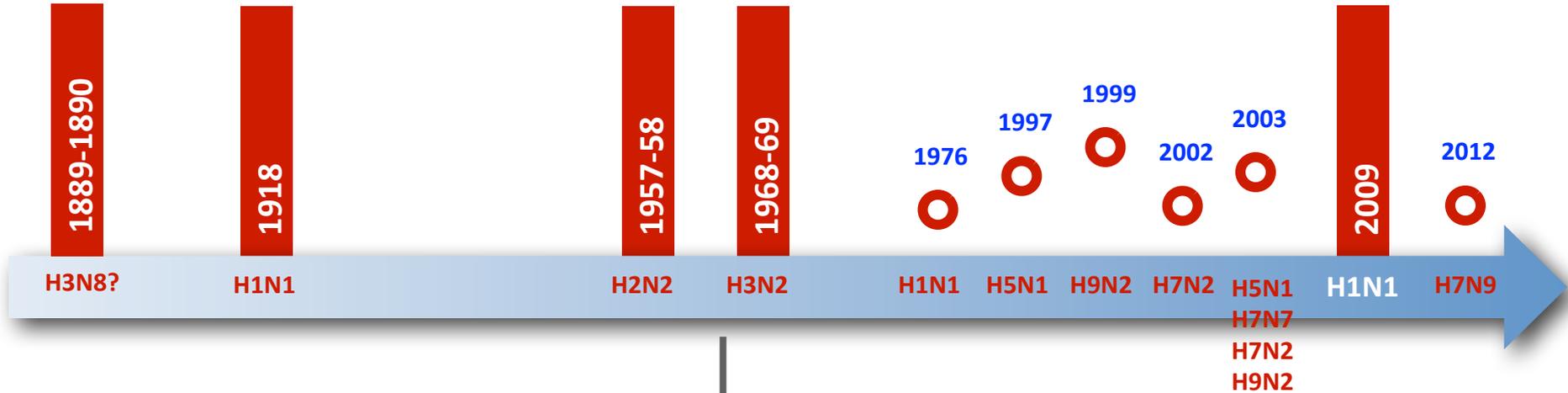


# infectious disease mortality (US)

Figure 4. Crude Mortality Rates for 10 Infectious Diseases

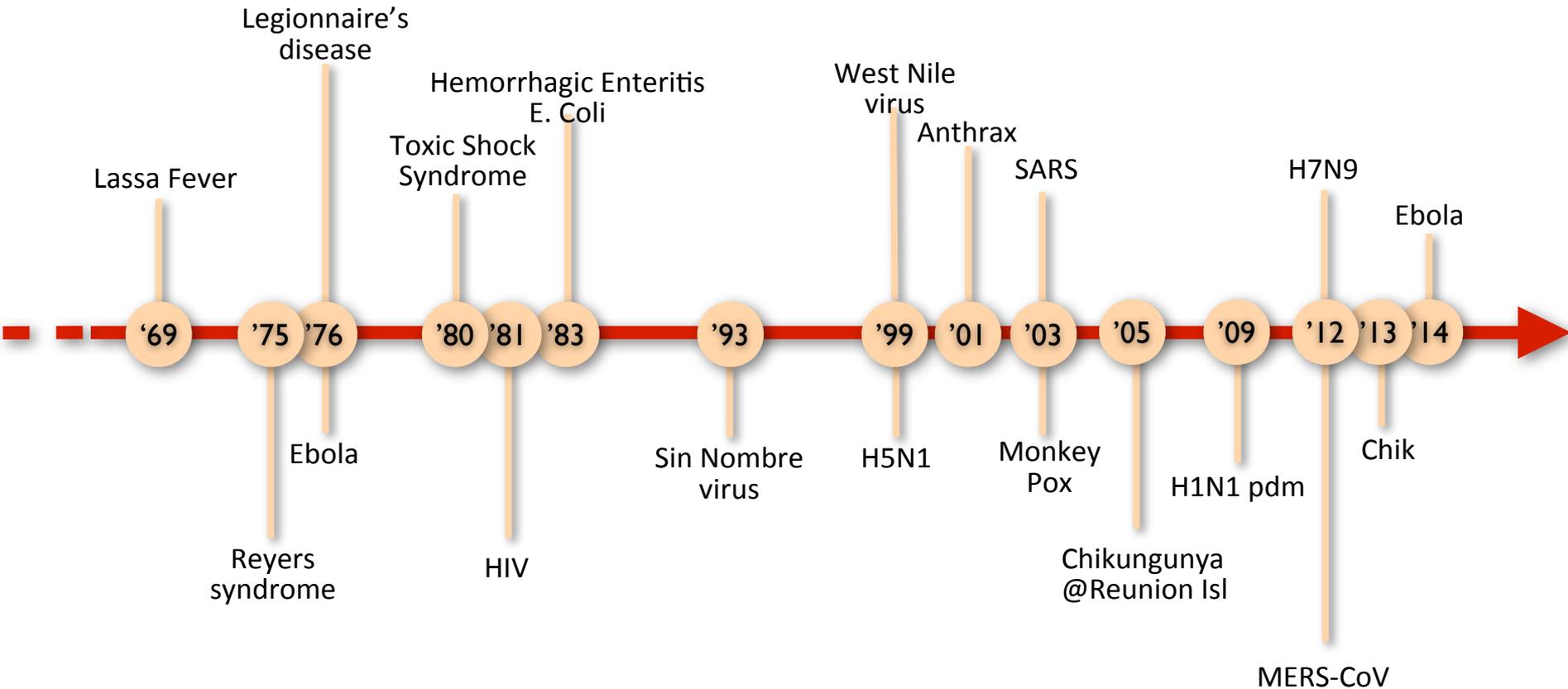


# brief history of flu



“because infectious diseases have been largely controlled in the US, we can now close the book on infectious diseases”  
US Surgeon General 1967

# some recent epidemics of EID



# some recent epidemics of EID

## HIV-related infectious diseases

1. *pneumocystis carinii* pneumonia
2. tuberculosis
3. mycobacterium-avium complex
4. Kaposi's sarcoma
5. HSV-2
6. cryptosporidium
7. microsporidium
8. *cryptococcus neoformans*
9. *penicillium marneffeii*
10. disseminated salmonella
11. bacillary angiomatosis
12. HPV
13. ...

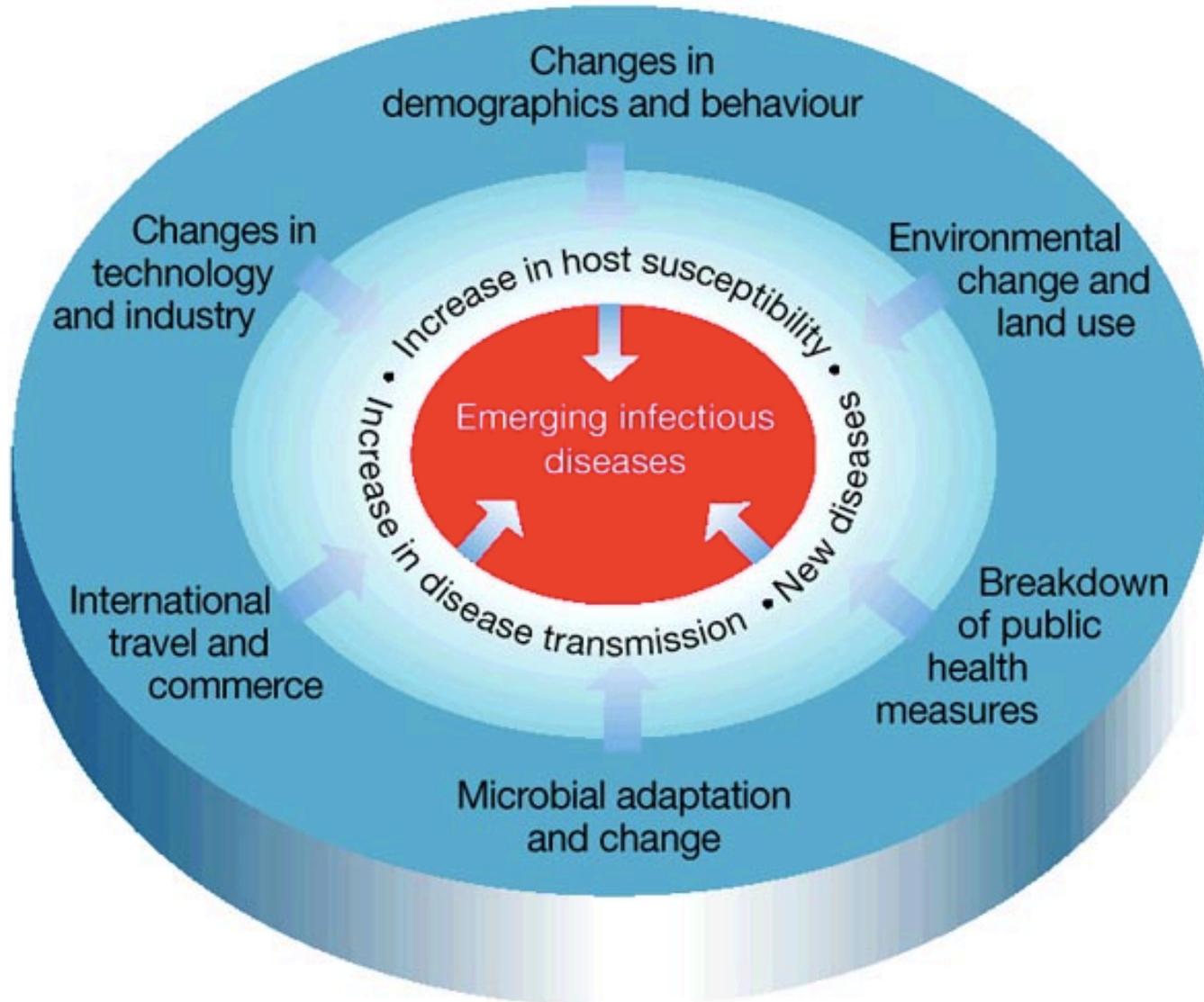
## non-HIV-related infectious diseases

1. SARS
2. influenza (H5N1, H1N1, H7N9, ...)
3. West Nile disease
4. variant CJD disease
5. monkeypox
6. Ebola and Marburg viruses
7. Dengue
8. Hanta virus
9. *E. coli*
10. antibiotic resistant microbes
11. Anthrax
12. Chikungunya
13. MERS-CoV
14. ...

# main factors leading to EID

- ▶ HIV
- ▶ population growth
- ▶ speed and ease of travel
- ▶ rural vs. urban
- ▶ climate change
- ▶ increased antibiotic use in humans and animals
- ▶ relocation of animals
- ▶ population aging
- ▶ human-animal contact

# main factors leading to EID



# >50% human pathogens from animals

disease	animal
HIV	primates
influenza	<i>many!</i> [originally derived from avian flu viruses]
ebola	primates
marburg	primates
hanta virus	deer mouse
arena virus	rodents
variant CJD	cattle
cryptosporidia	cattle
hendra virus	fruit bats
nipah virus	pigs, fruit bats
SARS	bats
monkey dogs	prairie dogs
MERS-CoV	camels?

# influenza

April 22nd 2013 - suspected mutation of an avian virus emerged outside Shanghai, China. Human fatality rate is unknown but out of 101 people infected, 20 have died.

 Pigs often a source of flu pandemics as they can be infected by bird, human & swine flus.

Worse-case, they act as a bridge for newly evolved virus strains to cross from birds to humans.

 The "Bird Flu" most mentioned in the media. Kills 60% of humans it infects. But direct human-to-human transmission has not been reported.

 Lesser known "bird flu" endemic in poultry in Eurasia. Rarely seen in humans.

 The most common variant of "Swine Flu". As "Spanish Flu" it killed 50-100 million people in 1918. The 2009-10 pandemic killed 15,000 worldwide.

 "Common flu" only found in humans. Less harmful than type A. Does not cause pandemics.

 Caused "Asian Flu" pandemic in 1957 then disappeared from human population. Still circulates in birds.

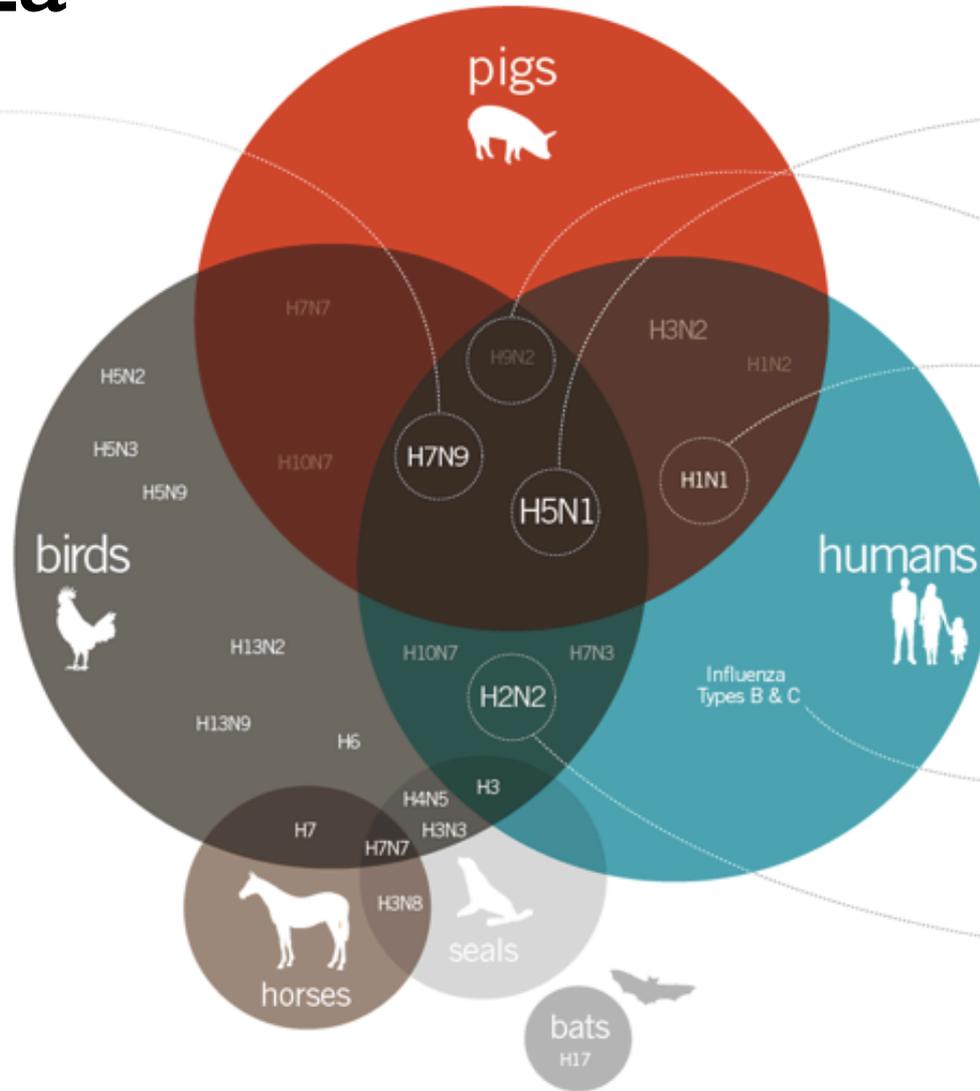
 Influenza Type A is divided into H & N strains (i.e. H1N1) referring to different combinations of:

**H** = hemagglutinin (binds to cells)

**N** = neuraminidase (surface enzyme)

text SIZE  
= human fatality rate

LIGHT TEXT  
= rarely infects humans



# reservoirs of IDs

human

animal

soil

water

# reservoirs of IDs

human

- ▶ HIV
- ▶ Hepatitis B virus
- ▶ Syphilis
- ▶ Gonorrhoea
- ▶ Typhoid fever
- ▶ Herpes simplex virus
- ▶ ...

animal

soil

water

# reservoirs of IDs

## human

- ▶ HIV
- ▶ Hepatitis B virus
- ▶ Syphilis
- ▶ Gonorrhoea
- ▶ Typhoid fever
- ▶ Herpes simplex virus
- ▶ ...

## soil

## animal

- ▶ rabies
- ▶ MERS-CoV
- ▶ influenza
- ▶ brucellosis
- ▶ anthrax
- ▶ ...

## water

# reservoirs of IDs

## human

- ▶ HIV
- ▶ Hepatitis B virus
- ▶ Syphilis
- ▶ Gonorrhoea
- ▶ Typhoid fever
- ▶ Herpes simplex virus
- ▶ ...

## animal

- ▶ rabies
- ▶ MERS-CoV
- ▶ influenza
- ▶ brucellosis
- ▶ anthrax
- ▶ ...

## soil

- ▶ tetanus
- ▶ botulism
- ▶ ...

## water

# reservoirs of IDs

## human

- ▶ HIV
- ▶ Hepatitis B virus
- ▶ Syphilis
- ▶ Gonorrhoea
- ▶ Typhoid fever
- ▶ Herpes simplex virus
- ▶ ...

## animal

- ▶ rabies
- ▶ MERS-CoV
- ▶ influenza
- ▶ brucellosis
- ▶ anthrax
- ▶ ...

## soil

- ▶ tetanus
- ▶ botulism
- ▶ ...

## water

- ▶ Legionnaires' disease
- ▶ ...

infectious diseases:  
some notation  
& classification

# example: 3 pathogens

influenza

- ◆ ...
- ◆ ...
- ◆ ...

HIV

- ◆ ...
- ◆ ...
- ◆ ...

tuberculosis

- ◆ ...
- ◆ ...
- ◆ ...

classification of the disease?

# IDs: a biological point of view

viruses

bacteria

fungi

parasites

# IDs: a biological point of view

viruses

- ▶ HIV
- ▶ influenza
- ▶ SARS, MERS-CoV
- ▶ chikungunya
- ▶ ...

bacteria

fungi

parasites

# IDs: a biological point of view

## viruses

- ▶ HIV
- ▶ influenza
- ▶ SARS, MERS-CoV
- ▶ chikungunya
- ▶ ...

## bacteria

- ▶ tuberculosis
- ▶ pneumonia
- ▶ salmonella
- ▶ tetanus
- ▶ typhoid fever
- ▶ ...

## fungi

## parasites

# IDs: a biological point of view

## viruses

- ▶ HIV
- ▶ influenza
- ▶ SARS, MERS-CoV
- ▶ chikungunya
- ▶ ...

## fungi

## bacteria

- ▶ tuberculosis
- ▶ pneumonia
- ▶ salmonella
- ▶ tetanus
- ▶ typhoid fever
- ▶ ...

## parasites

- ▶ malaria
- ▶ toxoplasmosis
- ▶ scabies
- ▶ ...

# IDs: a biological point of view

## viruses

- ▶ HIV
- ▶ influenza
- ▶ SARS, MERS-CoV
- ▶ chikungunya
- ▶ ...

## bacteria

- ▶ tuberculosis
- ▶ pneumonia
- ▶ salmonella
- ▶ tetanus
- ▶ typhoid fever
- ▶ ...

## fungi

- ▶ *Pneumocystis* pneumonia
- ▶ cryptococcal meningitis
- ▶ candidiasis
- ▶ ...

## parasites

- ▶ malaria
- ▶ toxoplasmosis
- ▶ scabies
- ▶ ...

# IDs: a clinician point of view

clinical symptoms

respiratory  
diseases

diarrheal  
diseases

cutaneous  
infection

central nervous  
system infection

fever of  
undet. origin

septicemic  
diseases

# IDs: an epidemiologist point of view

transmission

**contact**

- ▶ direct
- ▶ indirect (fomites, body secretions, ...)

**airborne**

- ▶ small-particle aerosols

**vector**

- ▶ anthropods

**food/water**

# IDs: an epidemiologist point of view

pathogen's properties

infectivity

pathogenicity

virulence

# IDs: an epidemiologist point of view

pathogen's properties

infectivity

▶ propensity for transmission

pathogenicity

virulence

# IDs: an epidemiologist point of view

pathogen's properties

**infectivity**

- ▶ propensity for transmission

**pathogenicity**

- ▶ propensity to cause disease or clinical symptoms

**virulence**

# IDs: an epidemiologist point of view

pathogen's properties

**infectivity**

- ▶ propensity for transmission

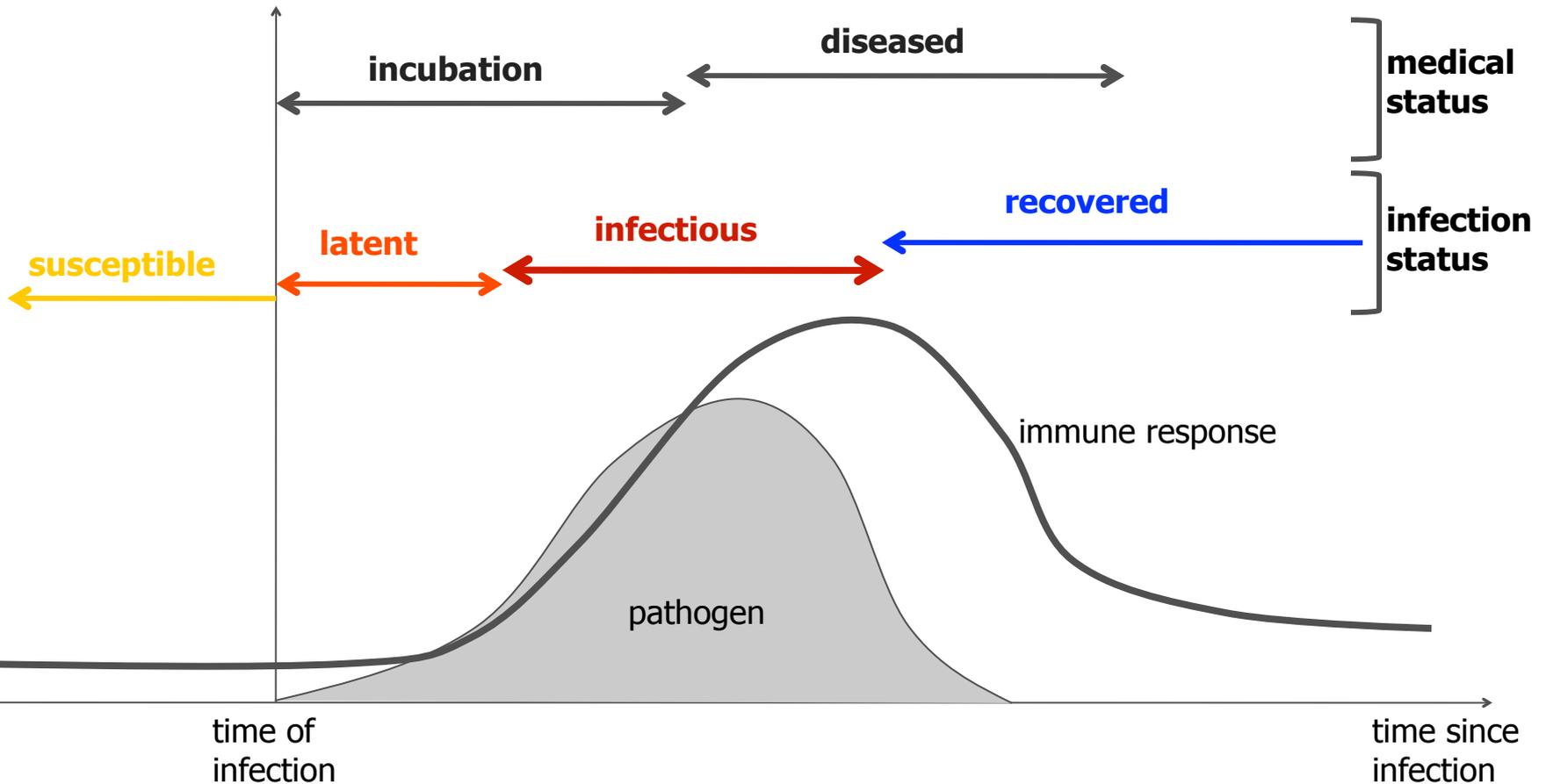
**pathogenicity**

- ▶ propensity to cause disease or clinical symptoms

**virulence**

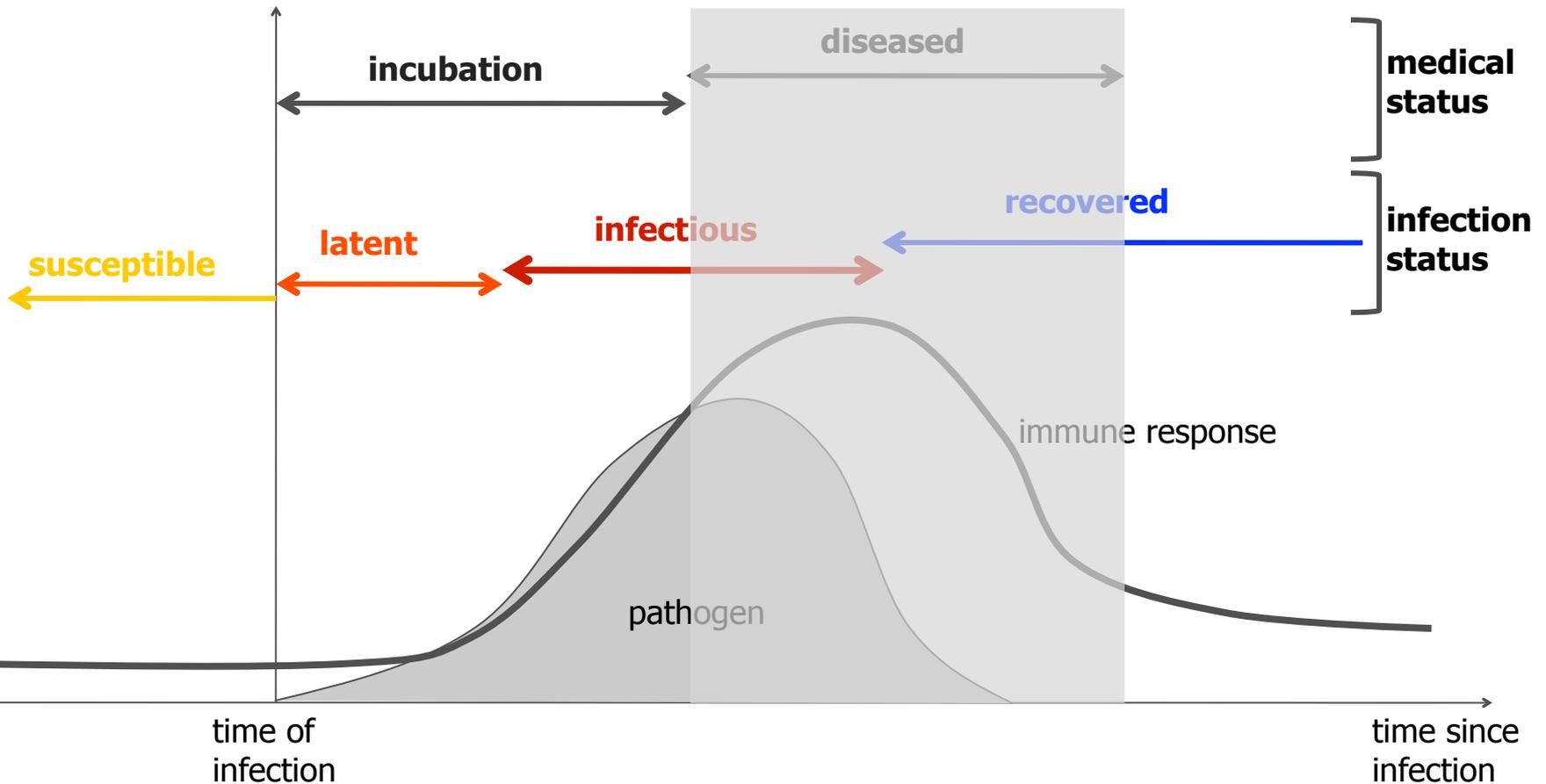
- ▶ propensity to cause severe disease

# epidemiologist vs. clinician point of view



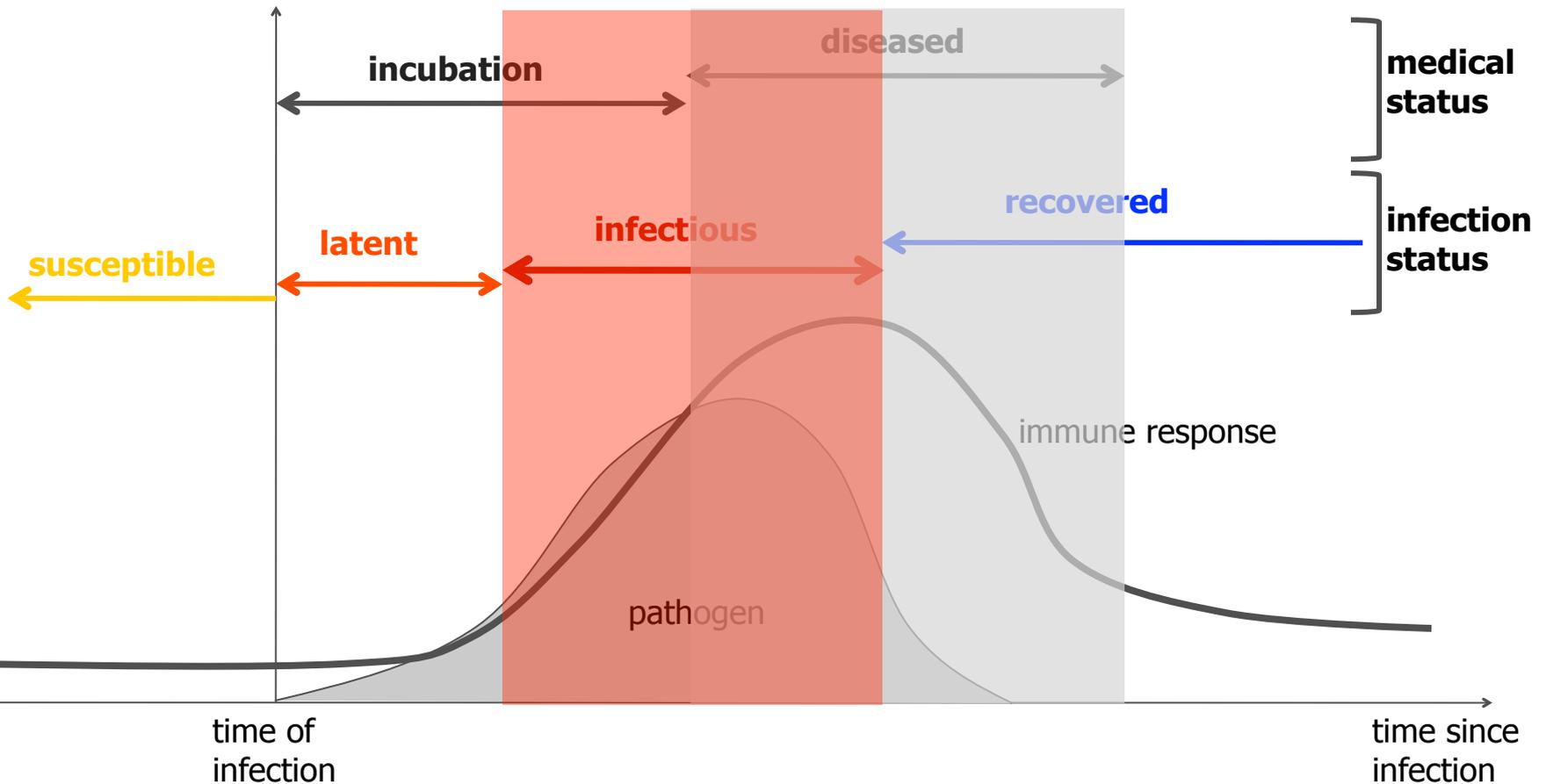
adapted from: Keeling & Rohani, Modeling Infectious Diseases (2008)

# epidemiologist vs. clinician point of view



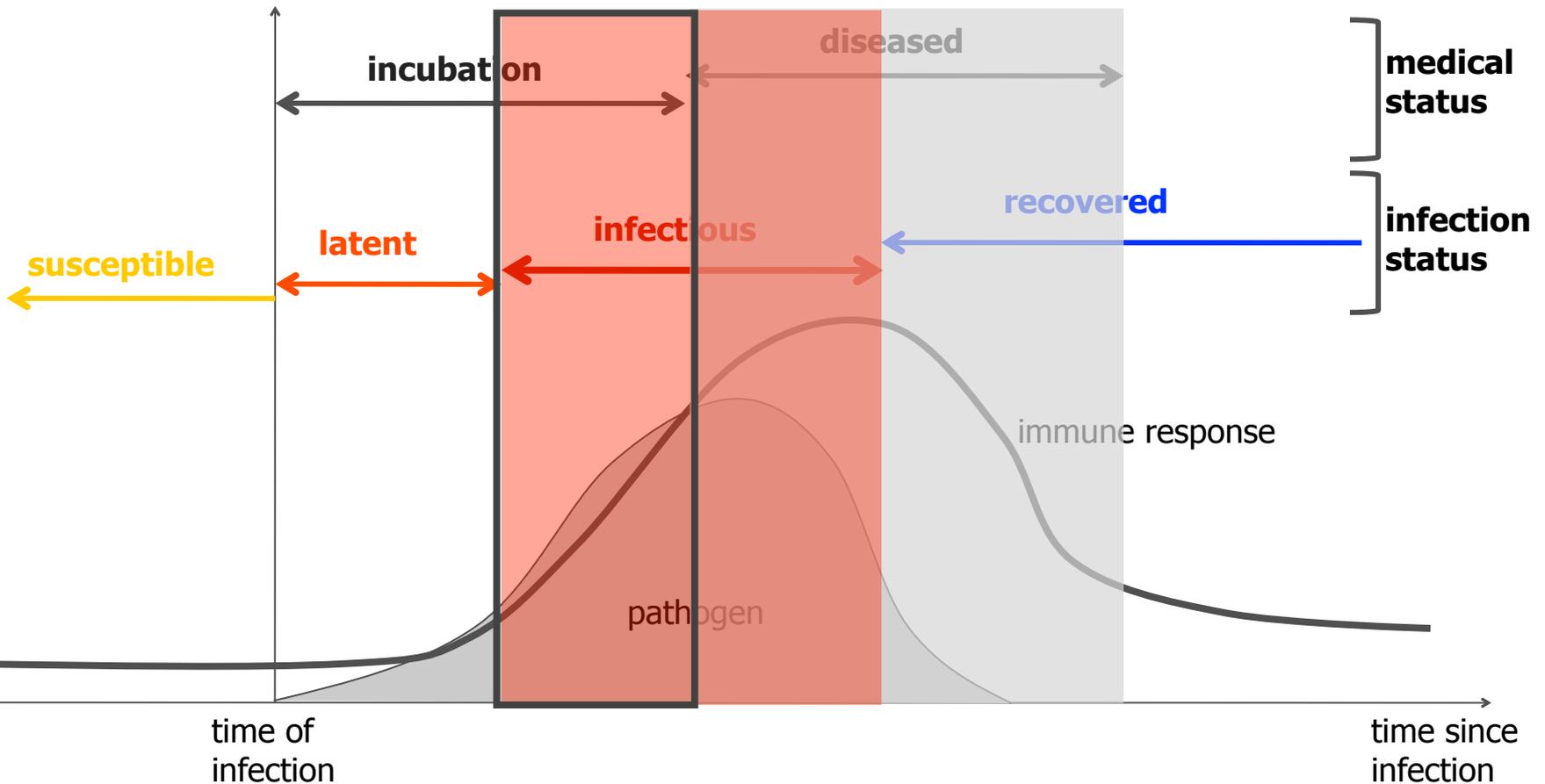
adapted from: Keeling & Rohani, Modeling Infectious Diseases (2008)

# epidemiologist vs. clinician point of view



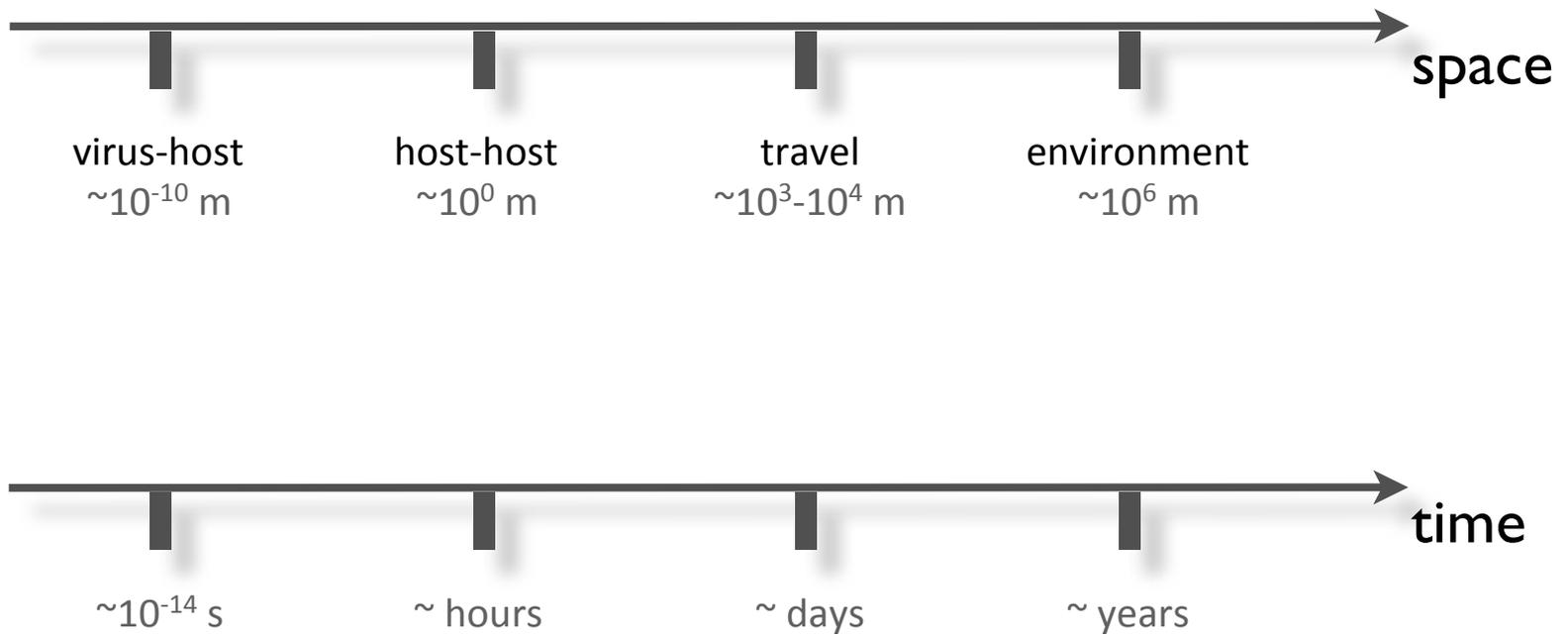
adapted from: Keeling & Rohani, Modeling Infectious Diseases (2008)

# epidemiologist vs. clinician point of view



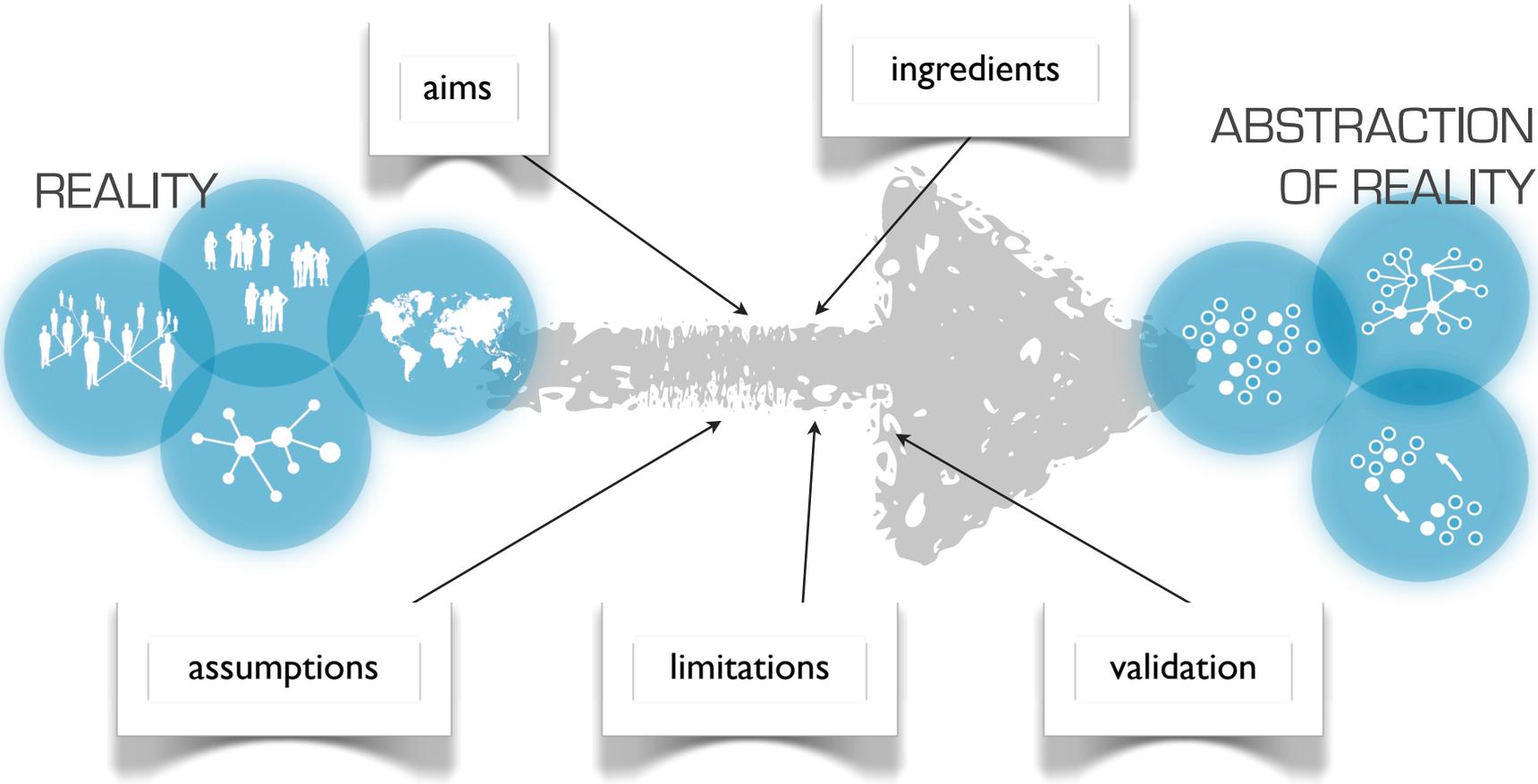
adapted from: Keeling & Rohani, Modeling Infectious Diseases (2008)

# scales



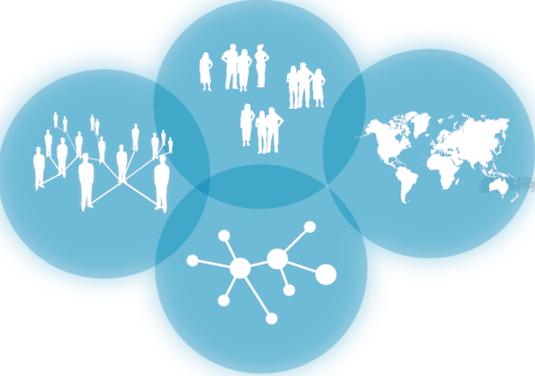
infectious diseases:  
simple models

# dynamical modeling 101

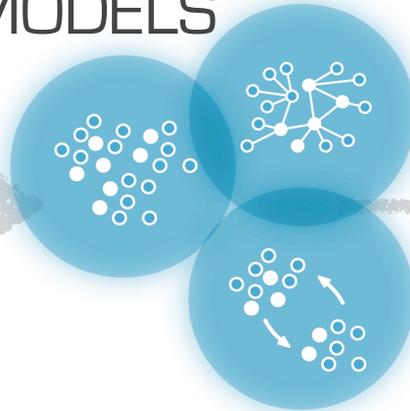


# aims

## REALITY

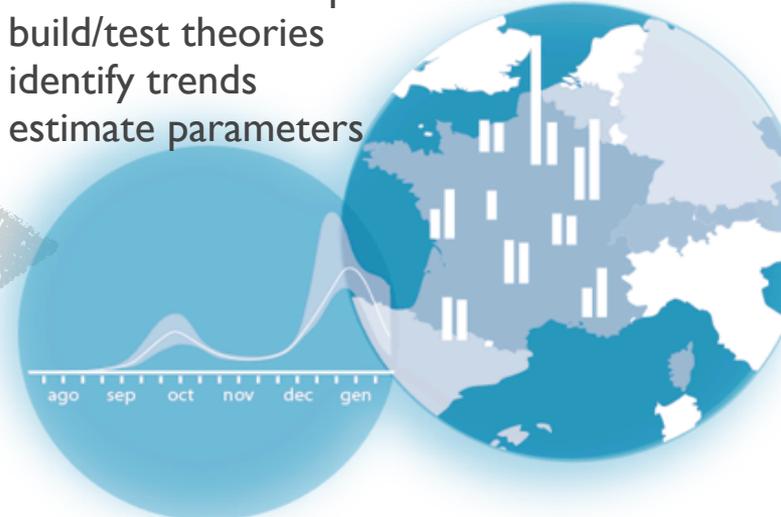


## MODELS



## UNDERSTAND

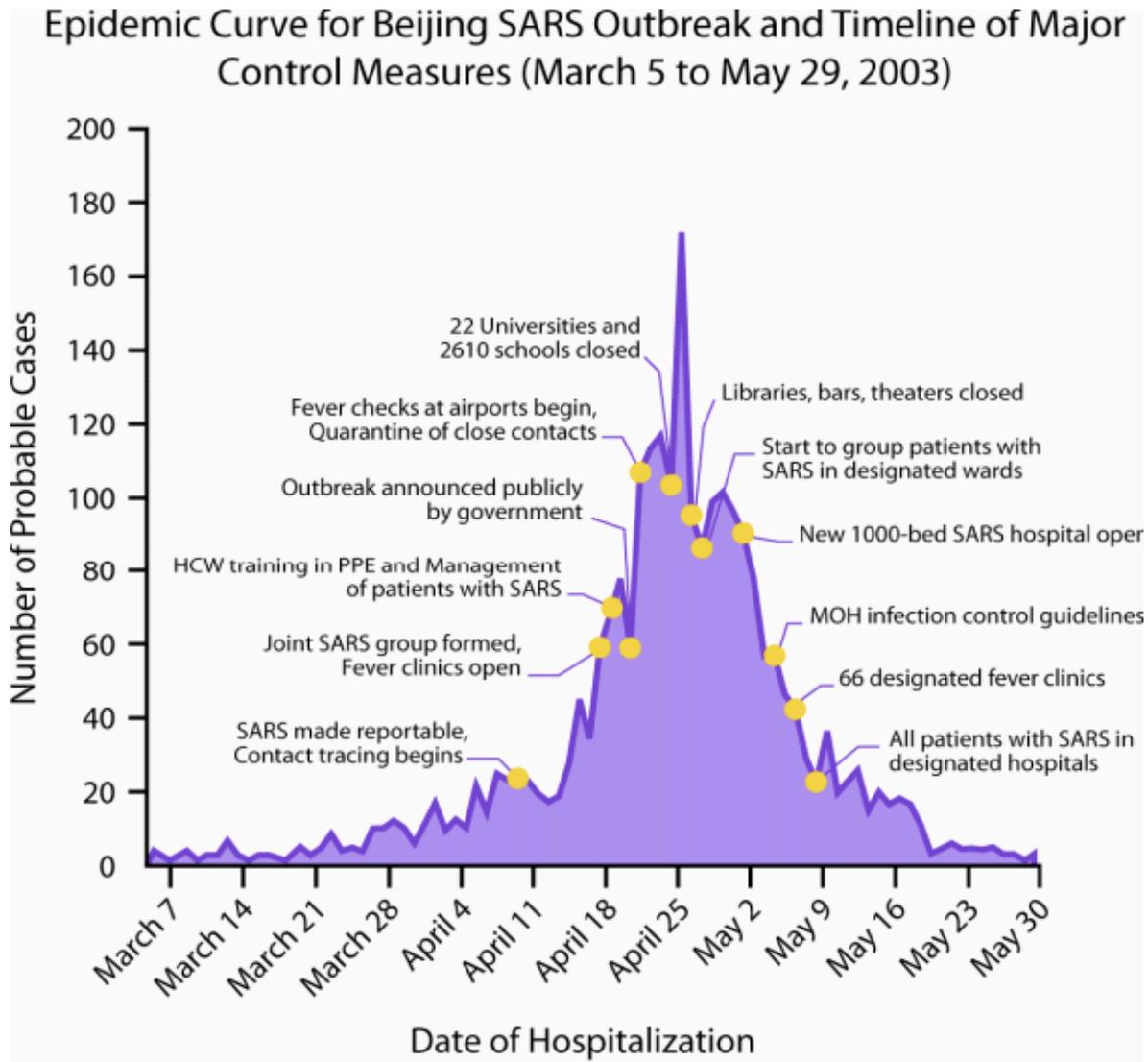
identify key mechanisms  
obtain basic conceptual results  
build/test theories  
identify trends  
estimate parameters



## PREPARE/PREDICT

explore scenarios  
test/evaluate/compare interventions  
answer specific questions  
provide forecasts (?)

# population level



# example: modeling 3 pathogens

influenza

- ◆ ...
- ◆ ...
- ◆ ...

HIV

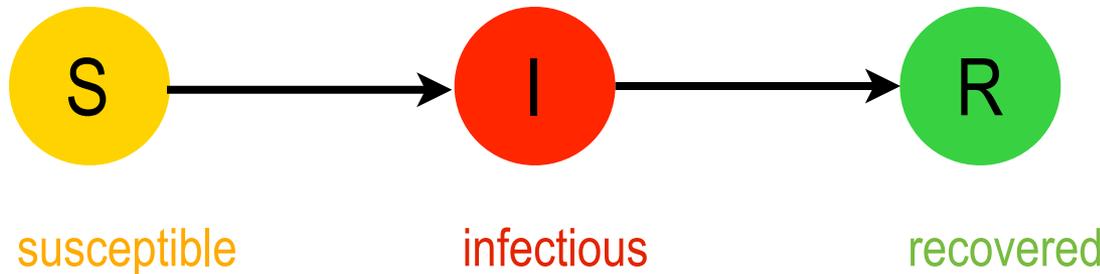
- ◆ ...
- ◆ ...
- ◆ ...

tuberculosis

- ◆ ...
- ◆ ...
- ◆ ...

ingredients? assumptions?

# simple compartmental models



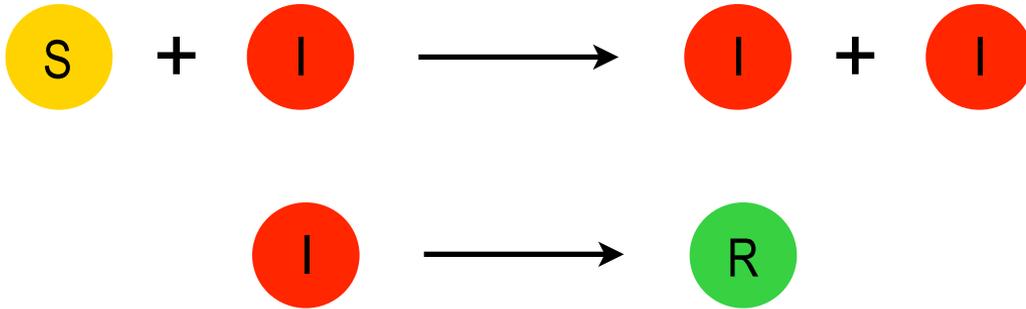
direct transmission

we neglect:

- age
- gender
- health
- job
- severity of disease
- environment
- social status
- latency
- susceptibility
- ...

# basic epidemic theory

chemical reactions

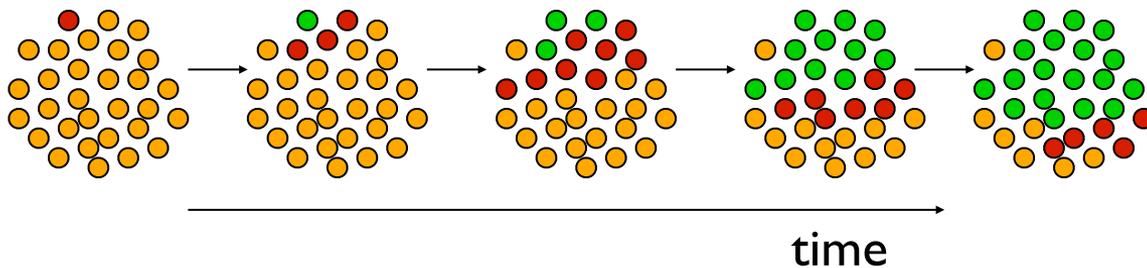


$$\frac{dS}{dt} = -\beta S \frac{I}{N}$$

$$\frac{dI}{dt} = \beta S \frac{I}{N} - \mu I$$

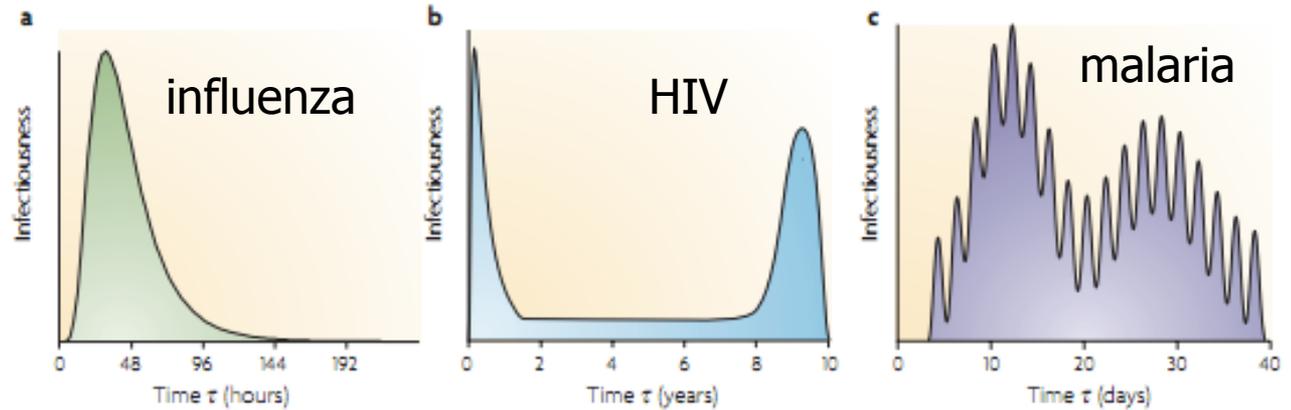
$$\frac{dR}{dt} = \mu I$$

homogeneous mixing



# infectiousness $\beta$

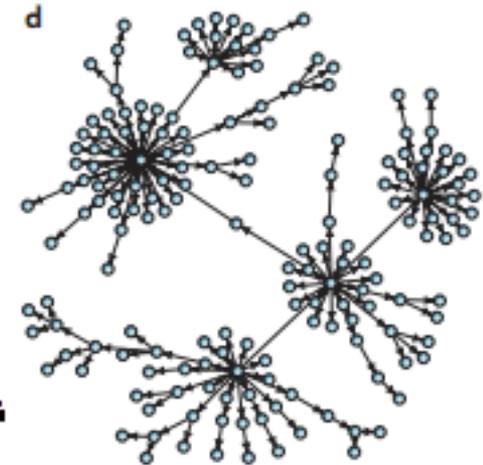
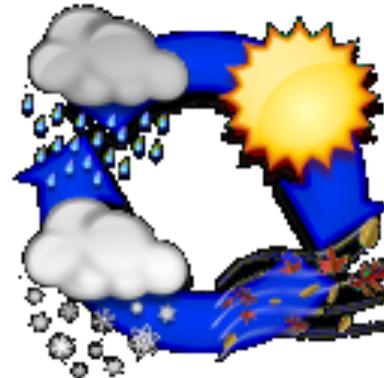
▪ **biological infectiousness:** related to pathogen's life cycle, host's immune system, interaction with drugs



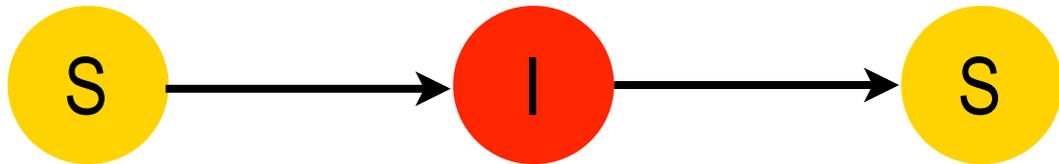
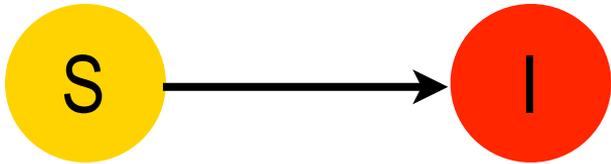
Grassly & Fraser, Nat Rev Microbiol (2008)

▪ **behavioral infectiousness:** related to the contact pattern of an infected individual

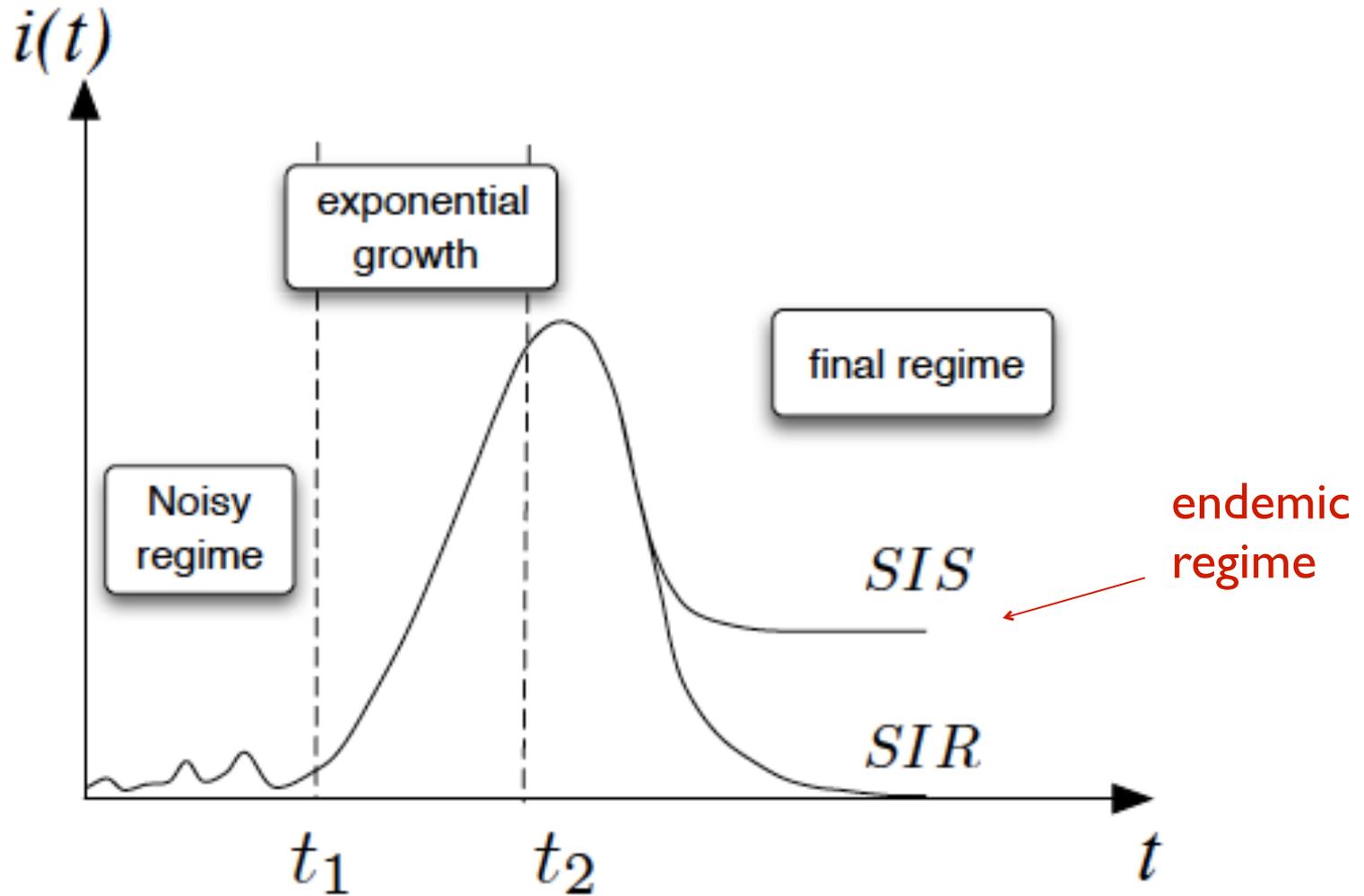
▪ **environmental infectiousness:** related to the location and environment of an infected individual, e.g. climatic and seasonal variations



# some basic variants



# at population level



# SIR model: basic questions

- how do we define a growing epidemics versus one that goes extinct?
- how strong does a virus need to be to propagate?

# SIR model: early stage

$$\frac{dI}{dt} = \lambda \langle k \rangle S \frac{I}{N} - \mu I$$

new infections      loss of infectiousness

$\beta = \lambda \langle k \rangle$       average #contacts  
per-contact transmissibility

$$i = I/N \quad \text{prevalence}$$

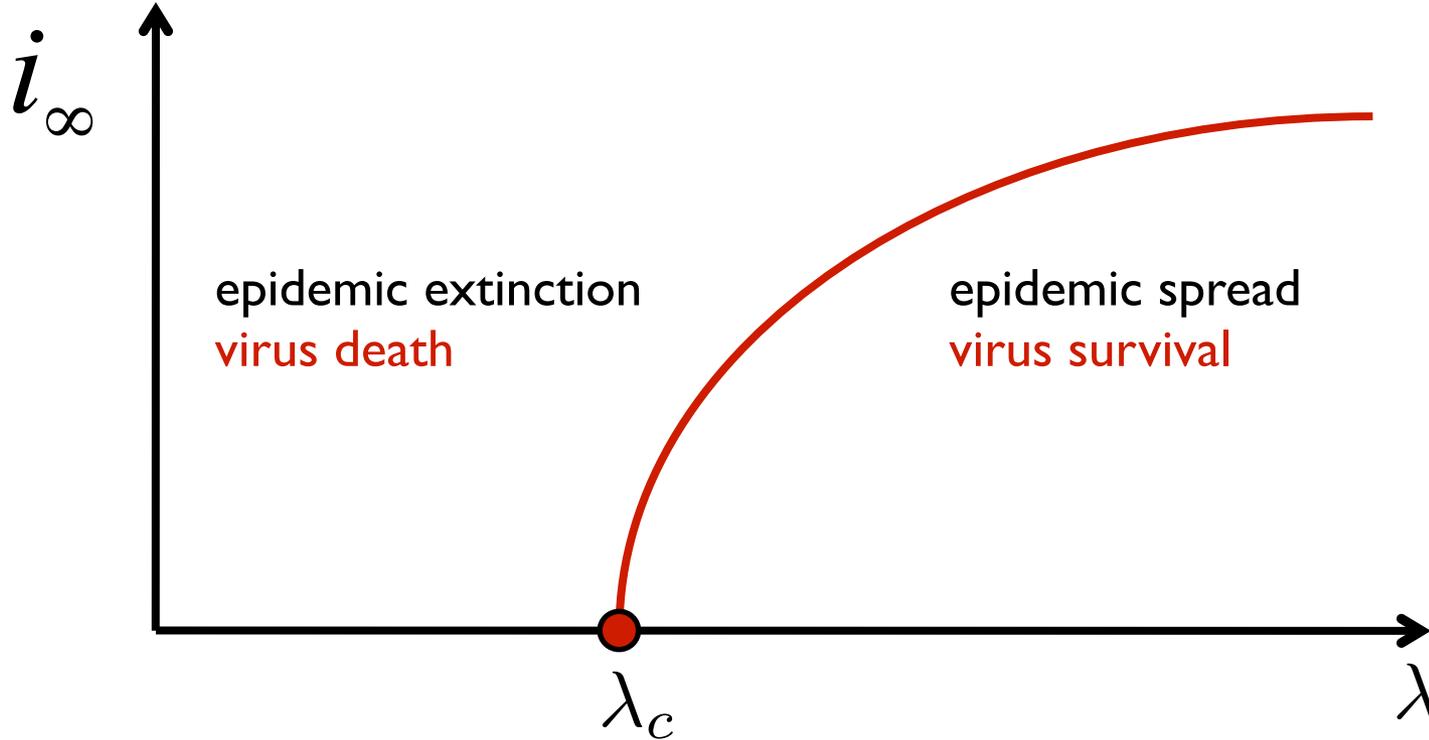
$$\frac{di}{dt} = \lambda \langle k \rangle s i - \mu i \simeq \lambda \langle k \rangle i - \mu i > 0 \Leftrightarrow \lambda \langle k \rangle - \mu > 0$$

$s \simeq 1$

$$\frac{di}{dt} > 0 \Leftrightarrow \lambda > \lambda_c = \mu / \langle k \rangle$$

epidemic threshold

# epidemic threshold



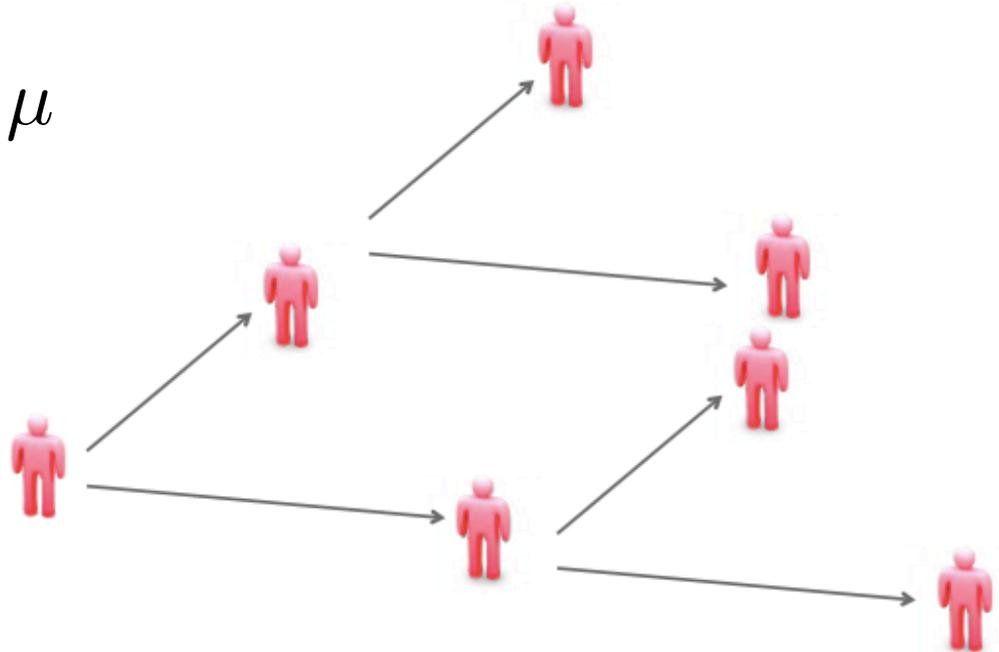
- very general result (SI, SIS, SEIR, ...)
- related to the **reproductive number**

# reproductive number

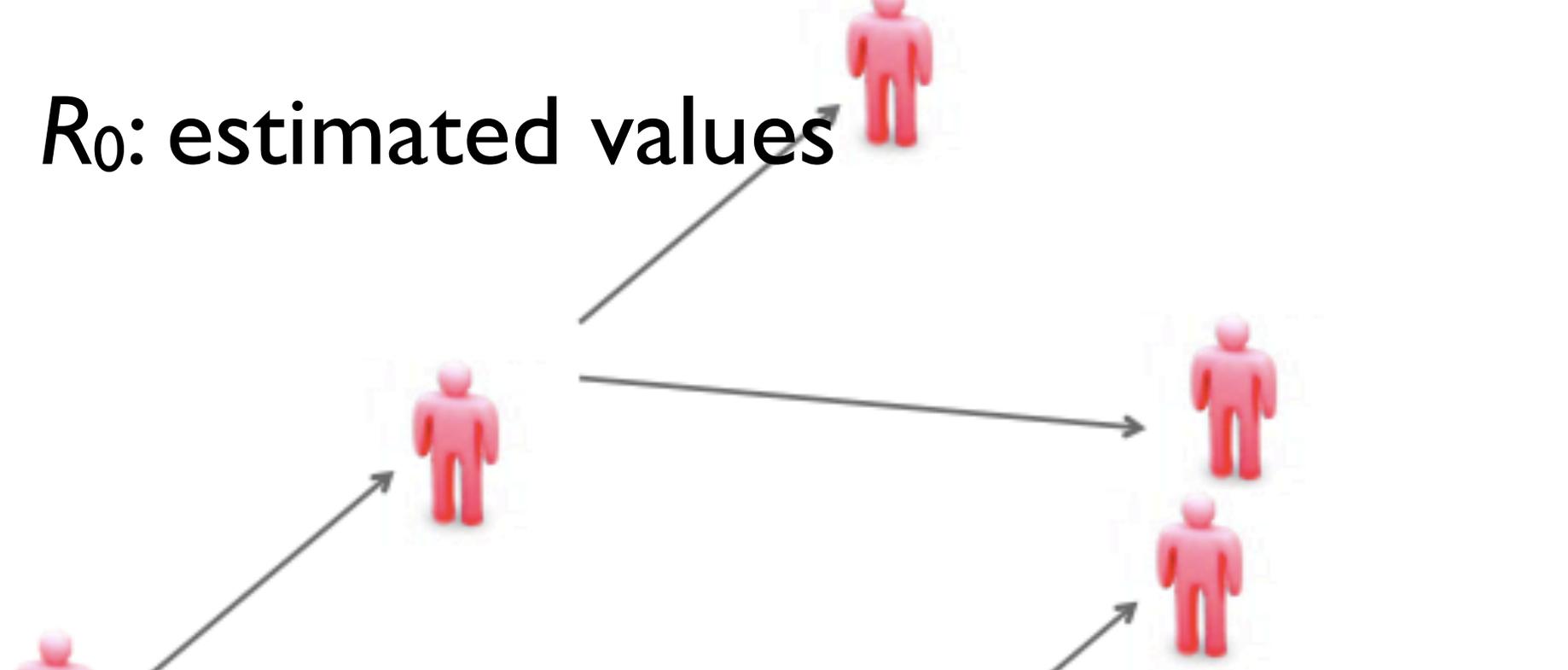
$R_0$  is the average number of individuals infected directly by an infected individual during his infectious period in a fully susceptible population.

$$\lambda > \lambda_c \iff \lambda \langle k \rangle > \mu$$

$$R_0 = \frac{\lambda \langle k \rangle}{\mu} > 1$$



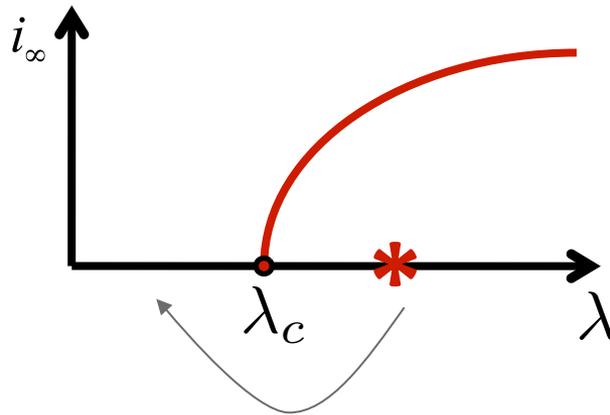
# $R_0$ : estimated values



infection	location	time period	$R_0$
measles	England & Wales	1950-68	16-18
	Ontario, Canada	1912-13	11-12
chicken pox	Eastern Nigeria	1960-68	16-17
	Maryland, USA	1913-17	7-8
	Baltimore, USA	1943	10-11
pertussis	England & Wales	1944-78	16-18

# epidemic threshold & health policy

if  $R_0 = \frac{\lambda \langle k \rangle}{\mu} > 1$  what can we do to stop/prevent an epidemic outbreak ?



we want to effectively reduce the epidemic below the threshold...

# again on the definition of $R_0$

*$R_0$  is the average number of individuals infected directly by an infected individual during his infectious period in a **fully susceptible** population.*

what if the population is **not fully susceptible**??

# again on the definition of $R_0$

$R_0$  is the average number of individuals infected directly by an infected individual during his infectious period in a **fully susceptible** population.

what if the population is **not fully susceptible**??

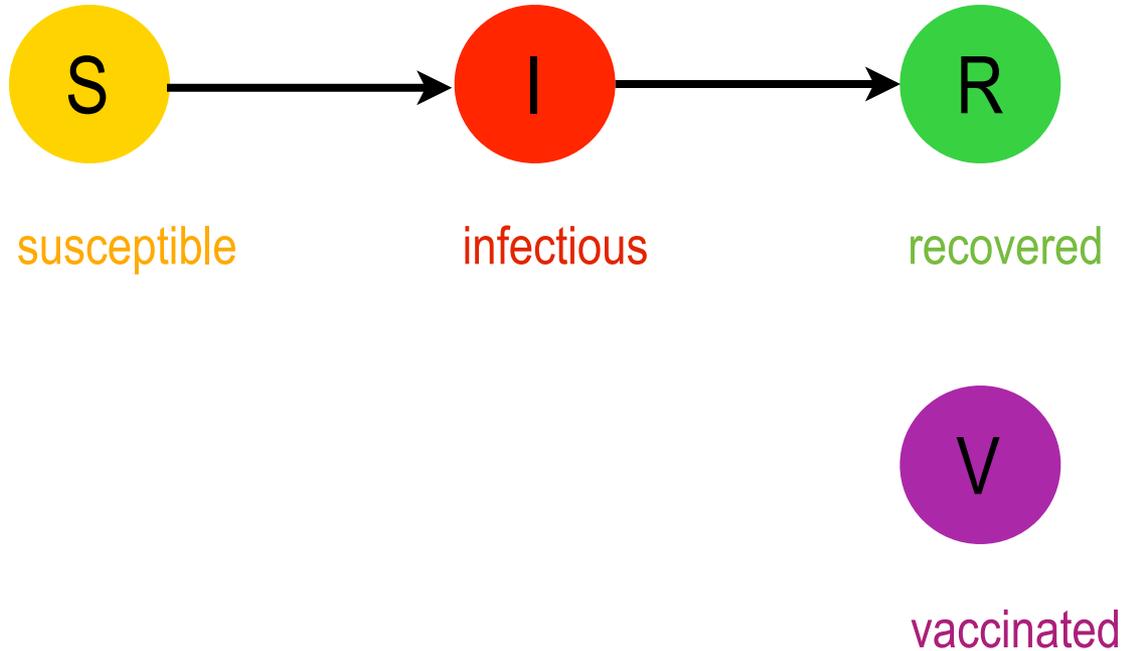
...for example... *vaccination*



a certain fraction of the population is vaccinated and therefore immune to the disease

► additional compartment: **vaccinated**

# SIS epidemic with vaccination



$$N = S(t) + I(t) + R(t) + V \rightarrow S(t) = N - I(t) - R(t) - V$$

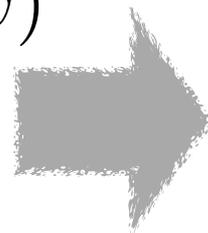
# SIR epidemic with vaccination

$$\frac{di}{dt} = \lambda \langle k \rangle (1 - v) i - \mu i, \quad v = \frac{V}{N}$$

$$\frac{di}{dt} > 0 \Leftrightarrow \lambda(1 - v) - \frac{\mu}{\langle k \rangle} > 0$$

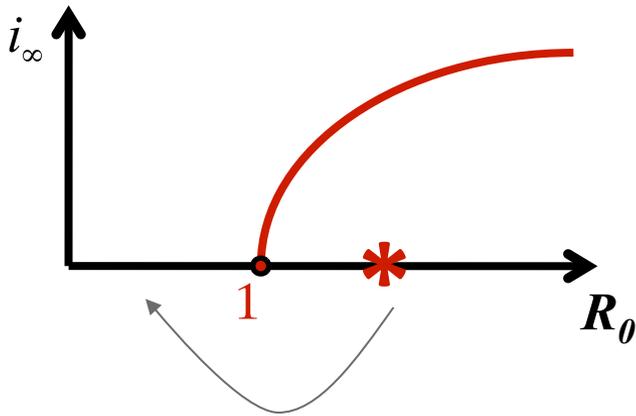
$$\boxed{\lambda(1 - v)} - \lambda_c > 0$$

$\lambda \Rightarrow \lambda(1 - v)$



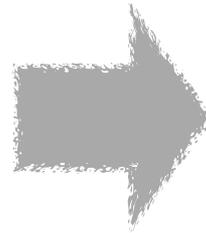
$$R'_0 = R_0(1 - v) < R_0$$

# immunization threshold



condition to prevent/control  
an epidemic outbreak ?

$v_c$  such that  $R'_0 < 1$



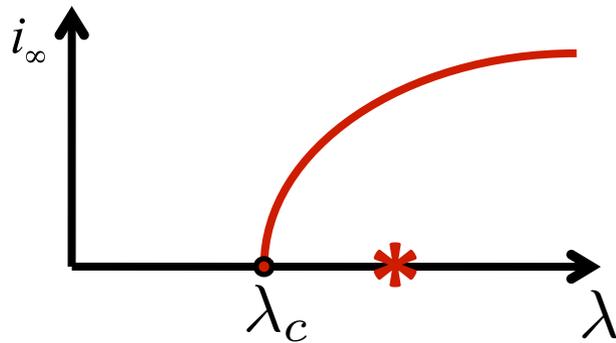
$$R'_0 = R_0(1 - v) < 1$$

$$v > v_c = \frac{R_0 - 1}{R_0}$$

fraction of the population to be  
vaccinated to prevent/control the  
outbreak

# simple epidemic models: *summary*

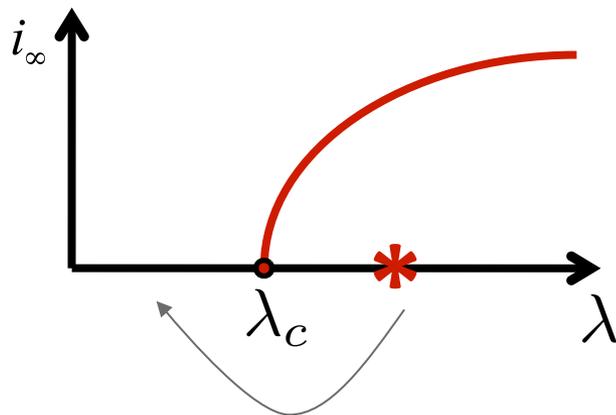
## homogeneous mixing



## epidemic threshold:

active phase i.e. virus spreads only if

$$\lambda > \lambda_c = \mu \langle k \rangle^{-1}$$

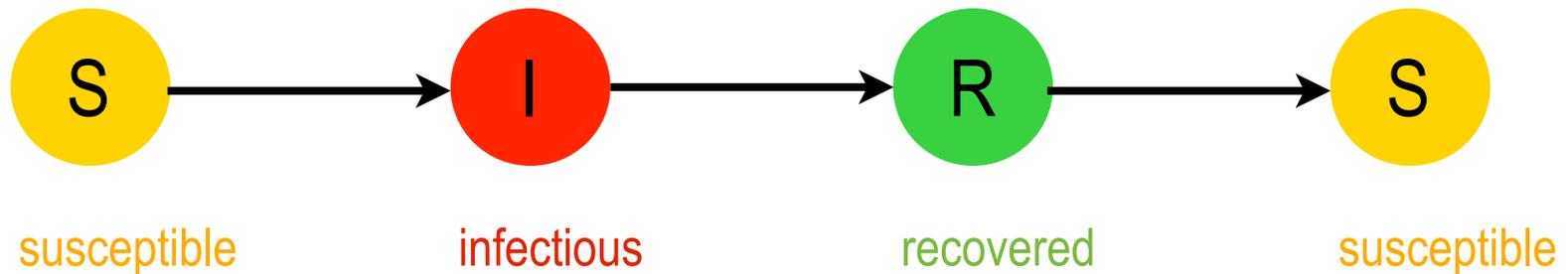
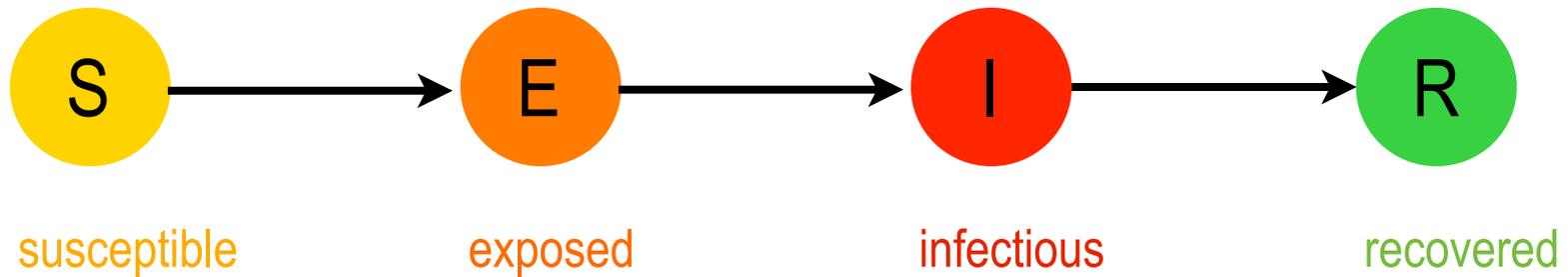


## immunization threshold:

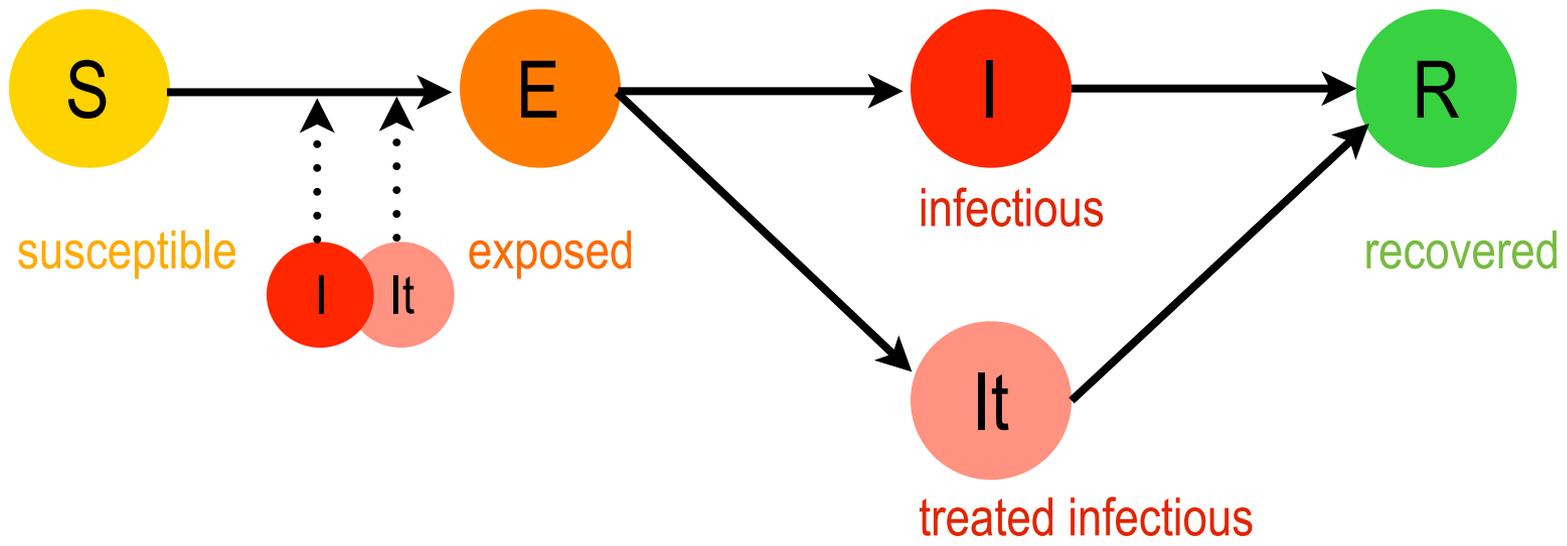
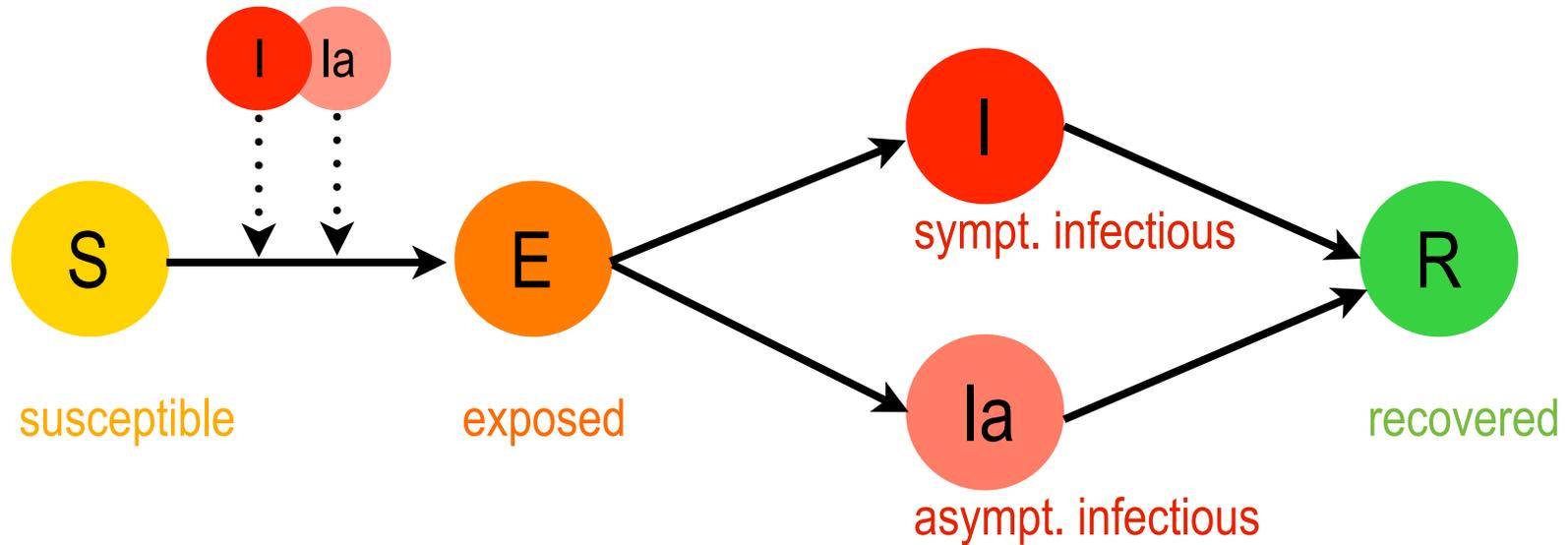
epidemic is stopped if

$$v > v_c = \frac{R_0 - 1}{R_0}$$

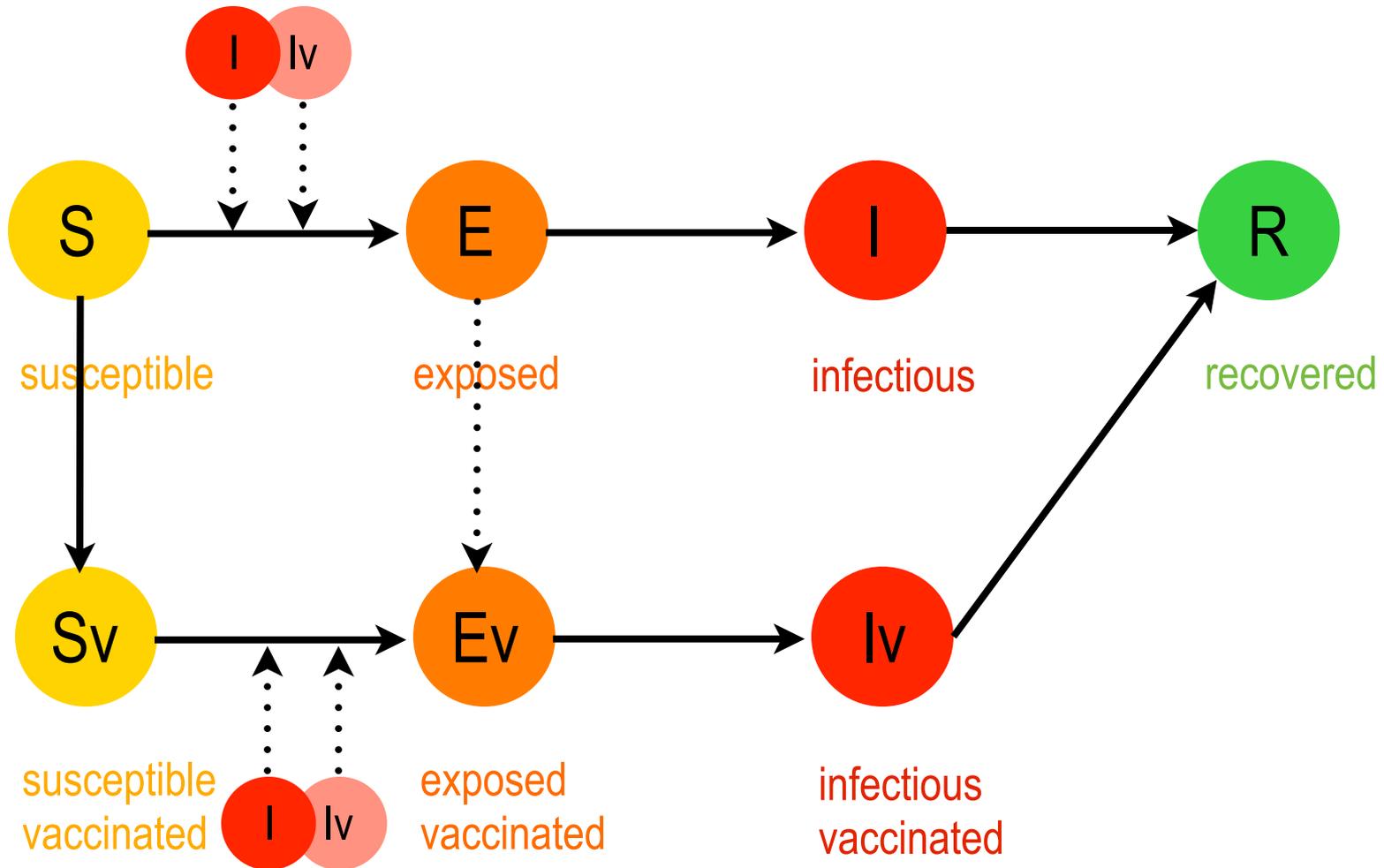
# going beyond (I): additional stages



# going beyond (I): additional stages



# going beyond (I): additional stages

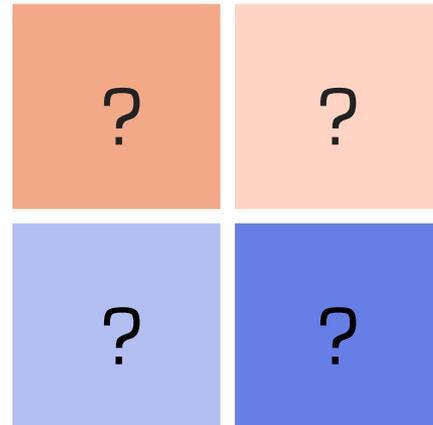


# going beyond (2): population classes

population profile



mixing



# going beyond (2): population classes

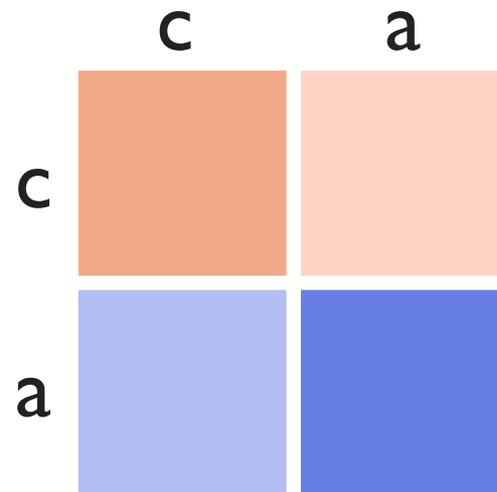
e.g.: flu

population profile

 children

 adults

mixing: **contacts**



# going beyond (2): population classes

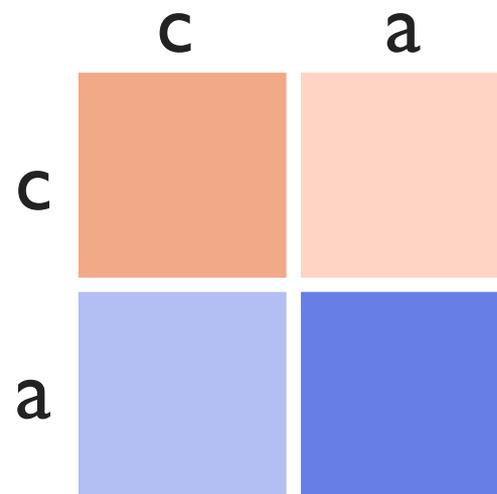
e.g.: HIV

population profile

● female

● male

mixing: **transmissibility**



# going beyond (3): frequency vs. density

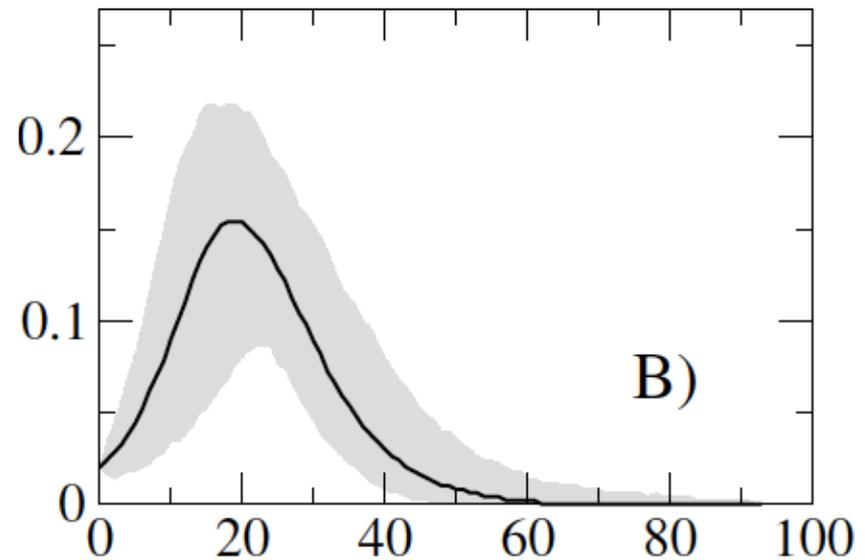
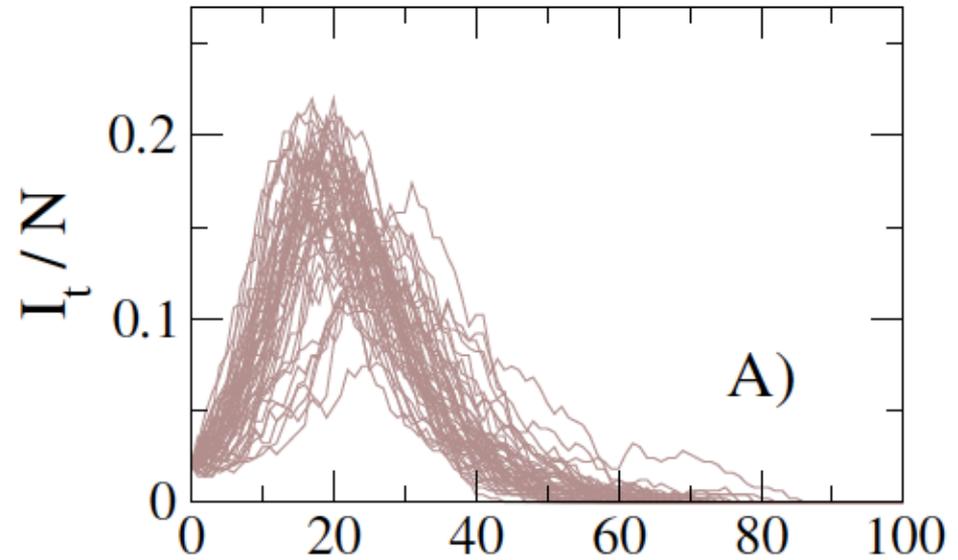
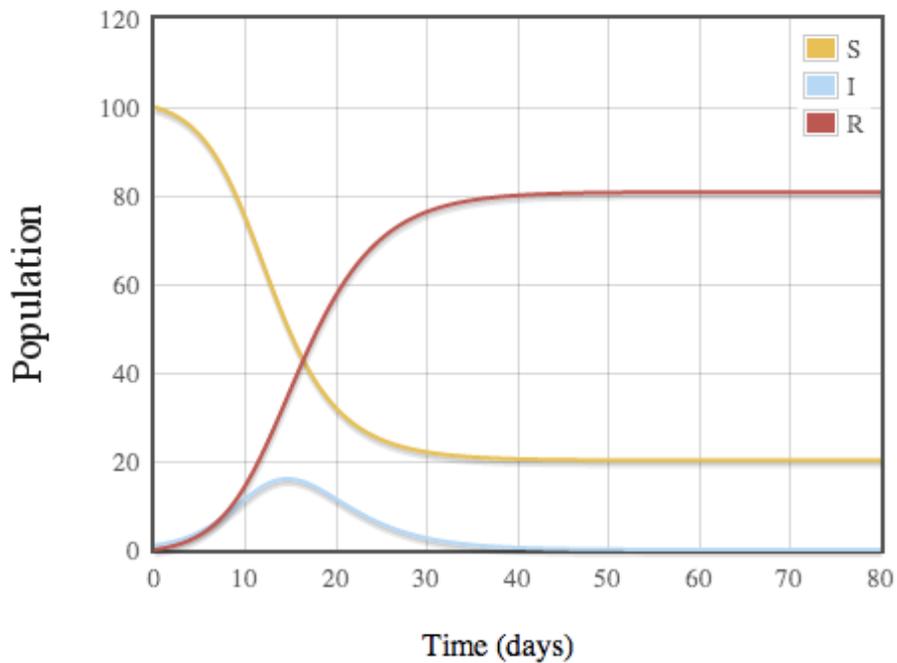
FD transmission: 
$$\frac{dI}{dt} = \beta S \frac{I}{N} - \mu I$$

DD transmission: 
$$\frac{dI}{dt} = \beta SI - \mu I$$

# going beyond (3): frequency vs. density

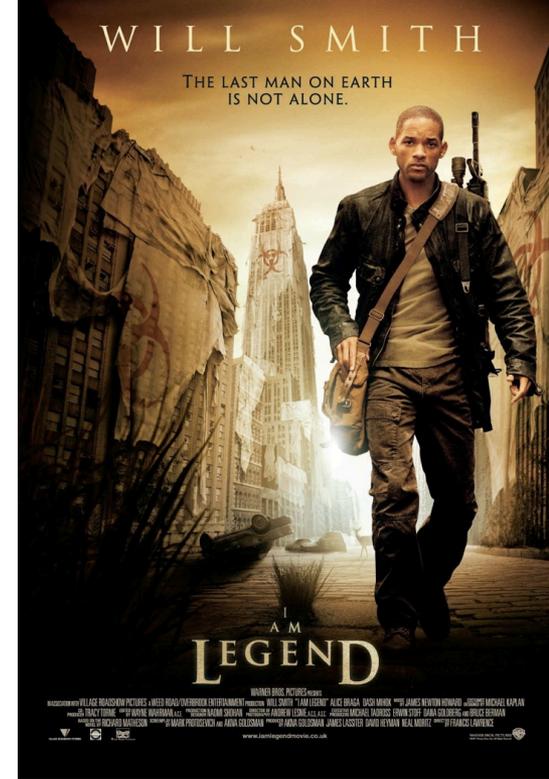


# going beyond (4): stochasticity



# going beyond (5): *I am legend*

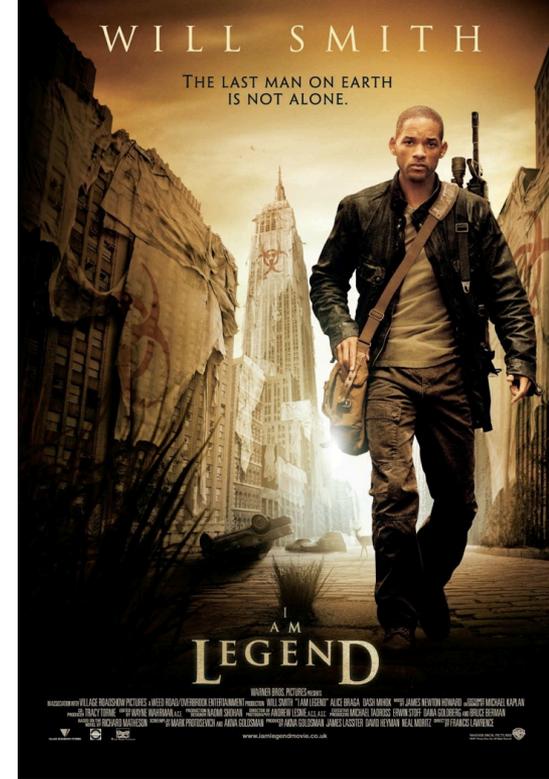
I. zombies (=infectious) work together to actively increase the number of zombies



# going beyond (5): *I am legend*

I. zombies (=infectious) work together to actively increase the number of zombies

$$\frac{di}{dt} = \beta s \sqrt{i} - \mu i$$

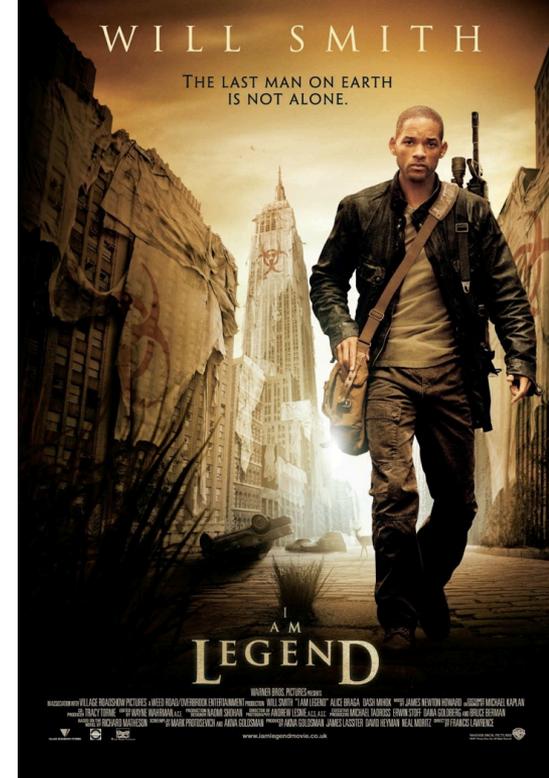


# going beyond (5): *I am legend*

1. zombies (=infectious) work together to actively increase the number of zombies

$$\frac{di}{dt} = \beta s \sqrt{i} - \mu i$$

2. zombies do not recover and stop infecting only when destroyed by a susceptible; susceptibles destroy as many zombies as they can.



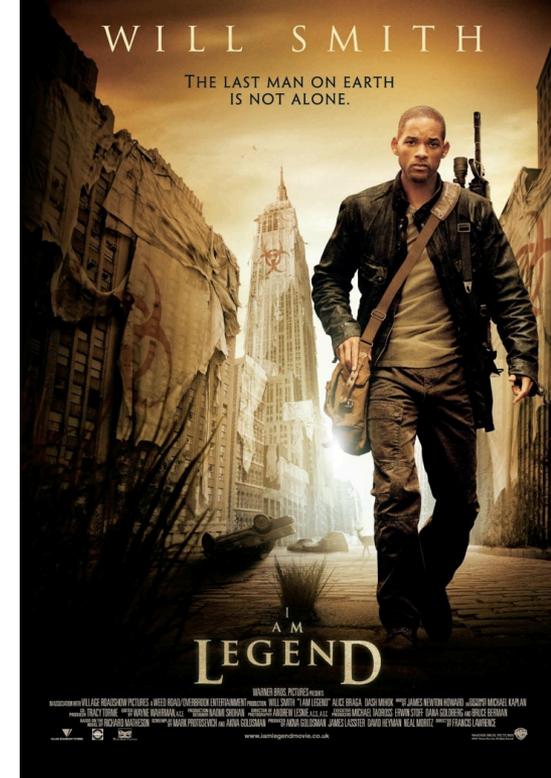
# going beyond (5): *I am legend*

1. zombies (=infectious) work together to actively increase the number of zombies

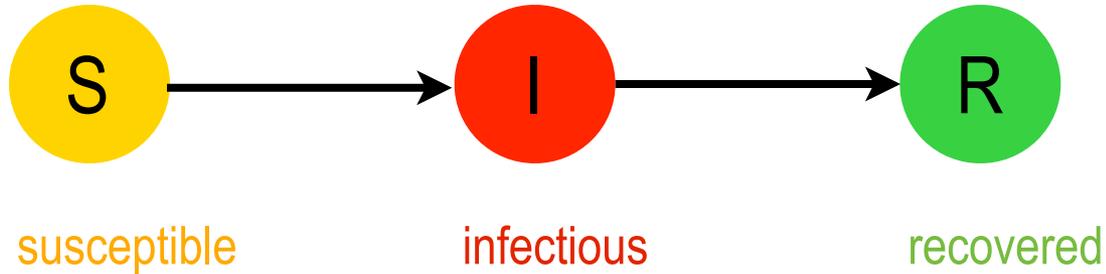
$$\frac{di}{dt} = \beta s \sqrt{i} - \mu i$$

2. zombies do not recover and stop infecting only when destroyed by a susceptible; susceptibles destroy as many zombies as they can.

$$\frac{di}{dt} = \beta s i - \mu s$$



# going beyond (6): heterogeneous contacts



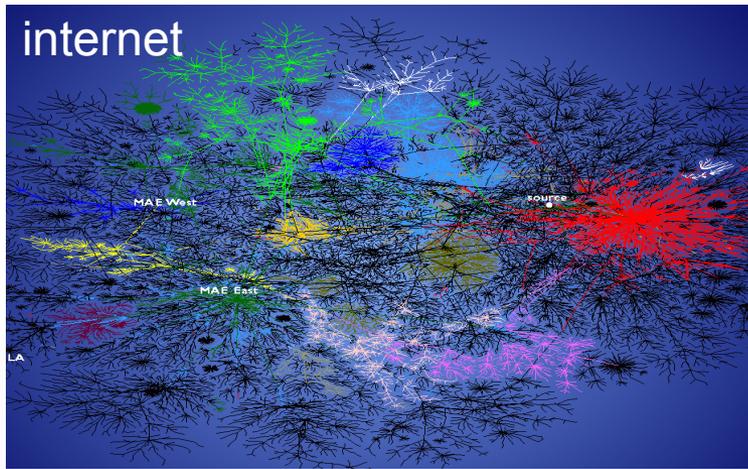
we neglect:

- age
- gender
- health
- job
- severity of disease
- environment
- social status
- latency
- susceptibility
- **contact heterogeneity**

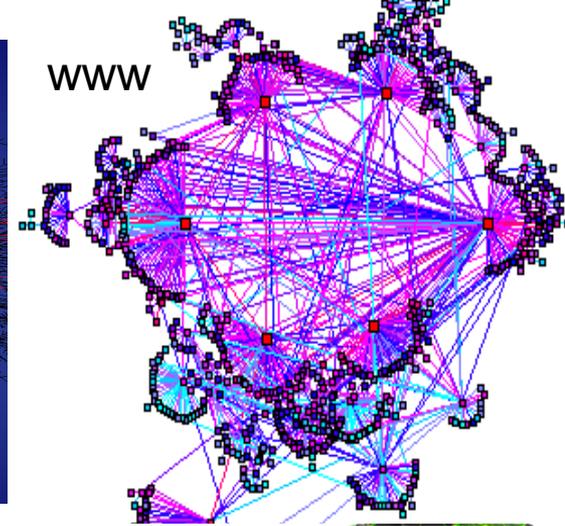
infectious diseases:  
contact network models

# networks

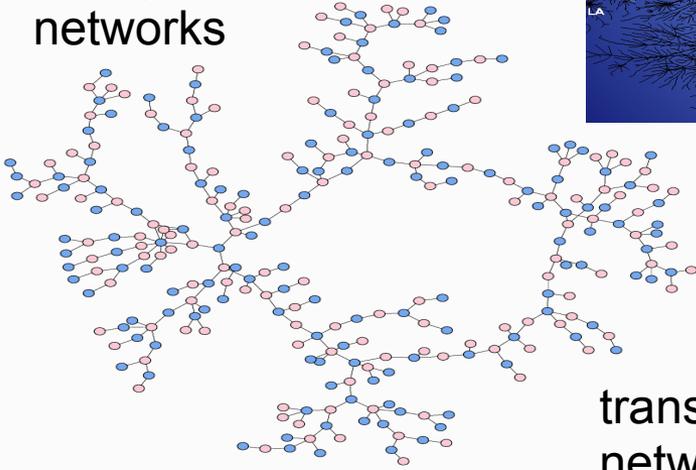
internet



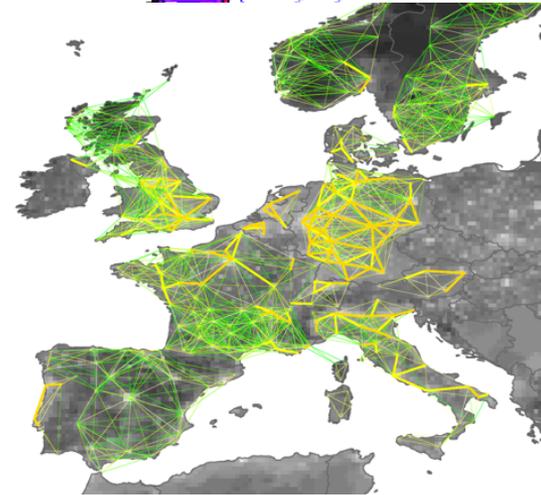
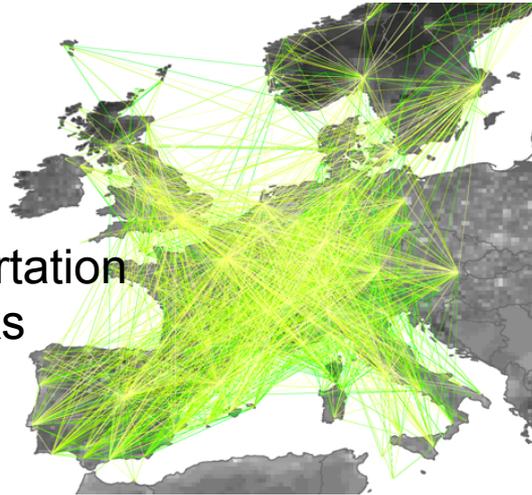
WWW



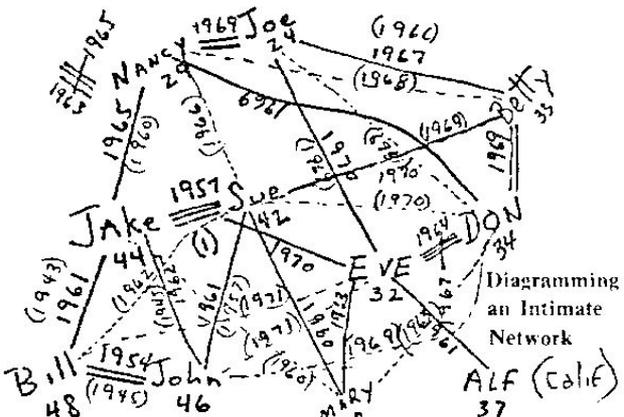
dating networks



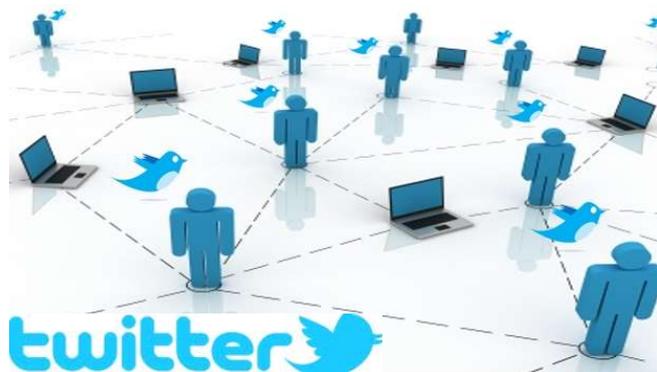
transportation networks



sexual networks



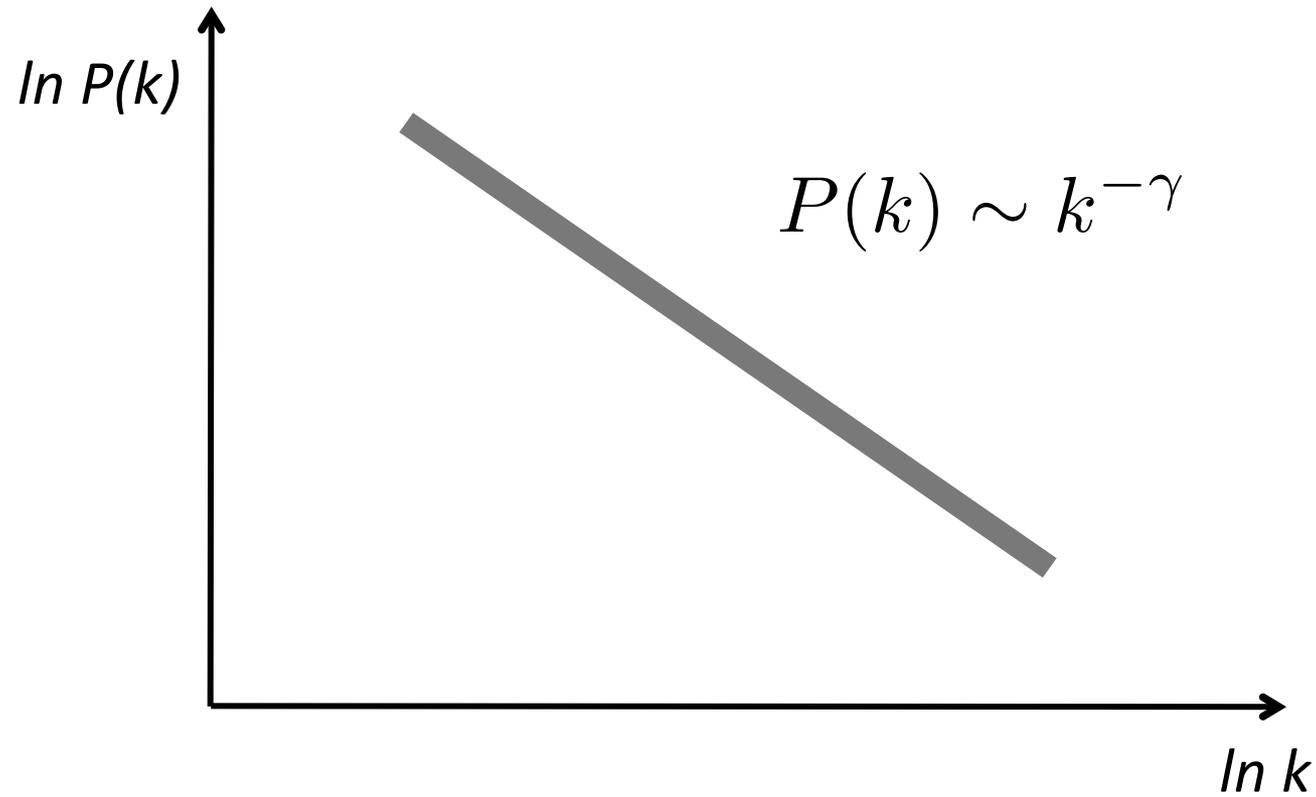
facebook



Facebook vous permet de rester en contact avec les personnes qui comptent dans votre vie.

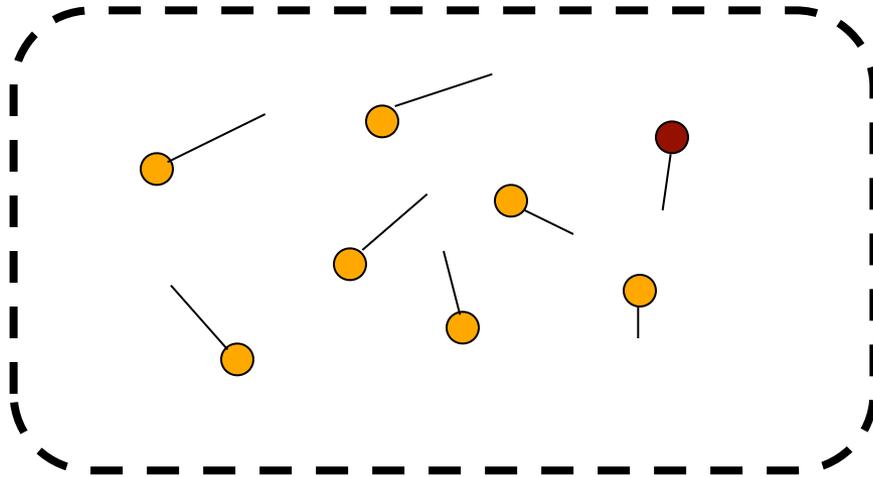


# heterogeneous networks

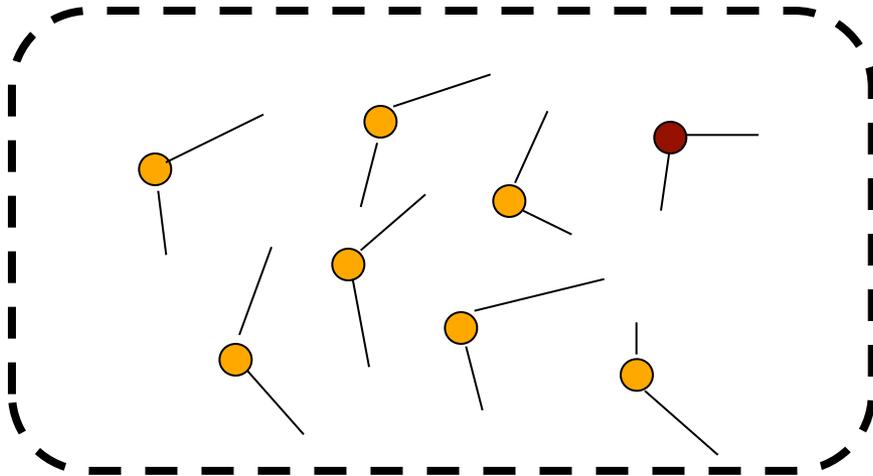




# degree-based representation or HMF



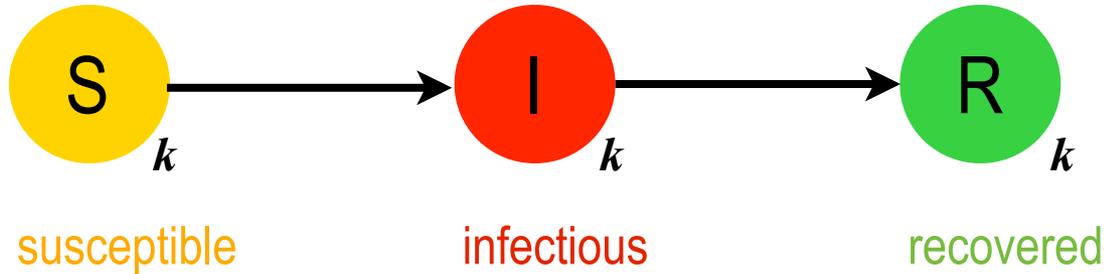
class of individuals with  
degree  $k=1$



class of individuals with  
degree  $k=2$

*etc.*

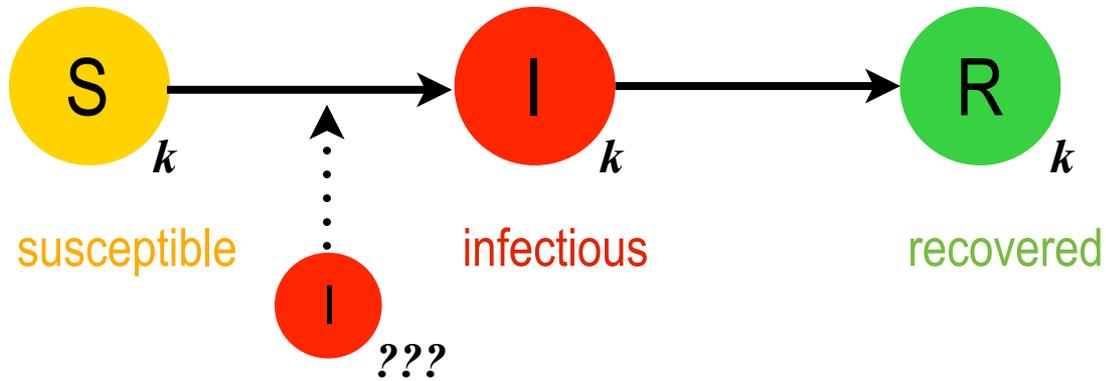
# degree-based representation



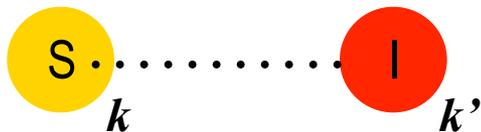
$$\frac{dI_k}{dt} = \lambda \langle k \rangle \frac{I}{N} S_k - \mu I_k$$

?

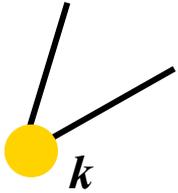
# contagion process



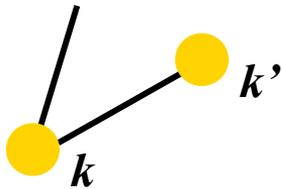
interaction with all possible classes  $k'$



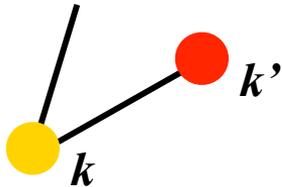
# components



number of possible contacts:  $k$

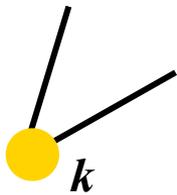
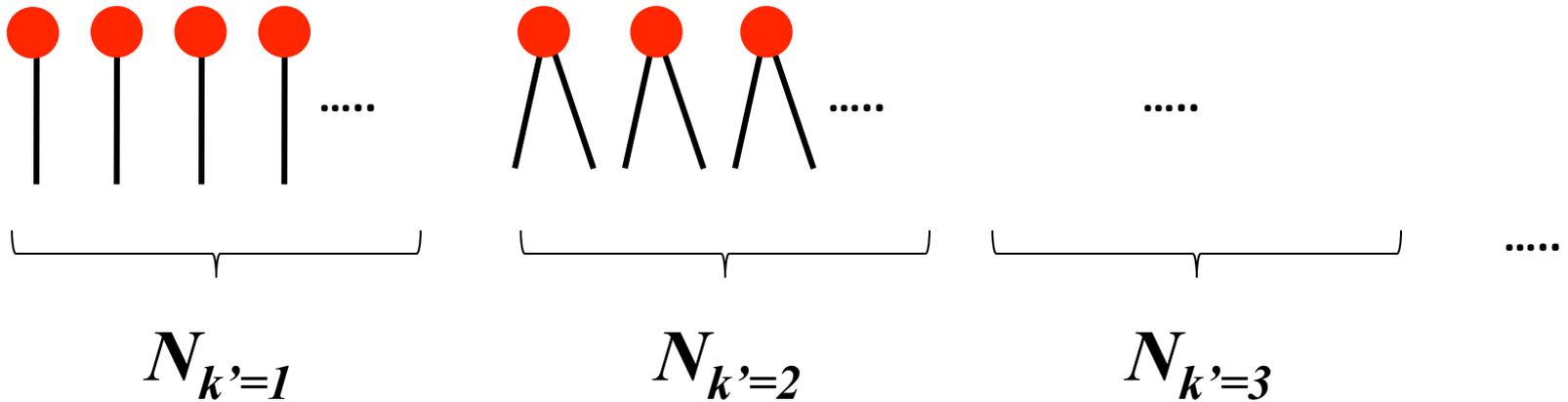


probability of contact with  $k'$ :  $P(k'|k)$



contact with  $k'$  is infectious:  $I_{k'}$

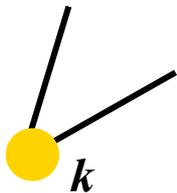
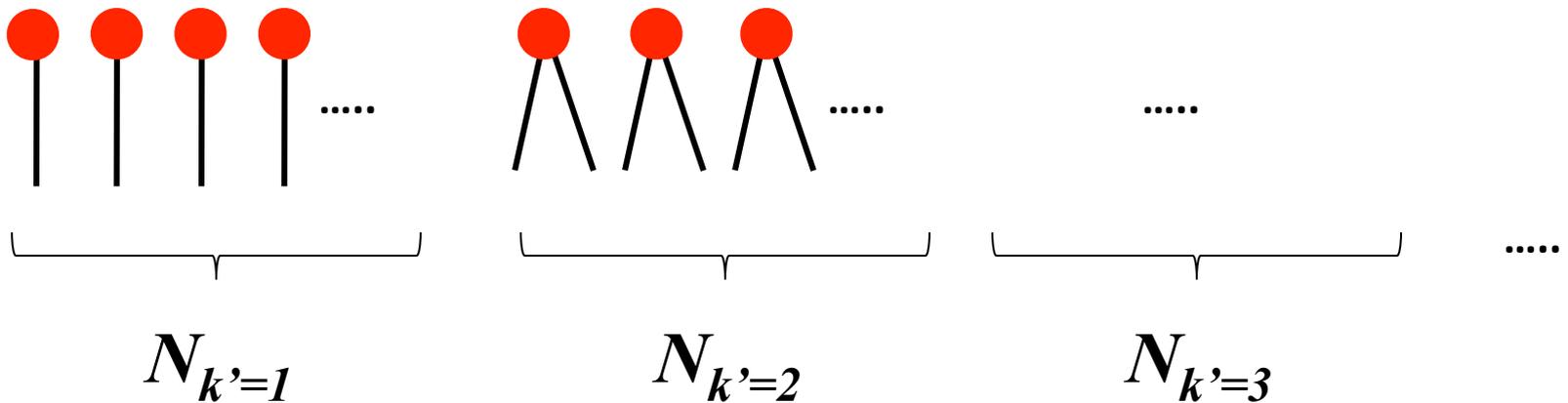
# probability that $k$ points to $k'$



$$P(1 | k) = \frac{1 \times N_1}{2E}$$

$$P(2 | k) = \frac{2 \times N_2}{2E}$$

# probability that $k$ points to $k'$



$$P(k'|k) = \frac{k' N_{k'}}{N \langle k \rangle} = \frac{k' P(k')}{\sum_{k'} k' P(k')} = \frac{k' P(k')}{\langle k \rangle}$$

assumption: *uncorrelated* network

# SIR on complex networks

$$\frac{dI}{dt} = \lambda k S_k \sum_{k'} P(k'|k) I_{k'} - \mu I_k$$

$$\frac{dI}{dt} > 0 \Leftrightarrow \lambda > \lambda_c = \frac{\mu \langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

epidemic threshold  
in heterogeneous  
networks

Cohen et al, PRL 2000  
Pastor-Satorras et al, PRL 2001  
Newman PRE 2002

$$\lambda > \lambda_c = \frac{\mu}{\langle k \rangle}$$

epidemic threshold  
in homogeneous  
mixing

# vanishing epidemic threshold

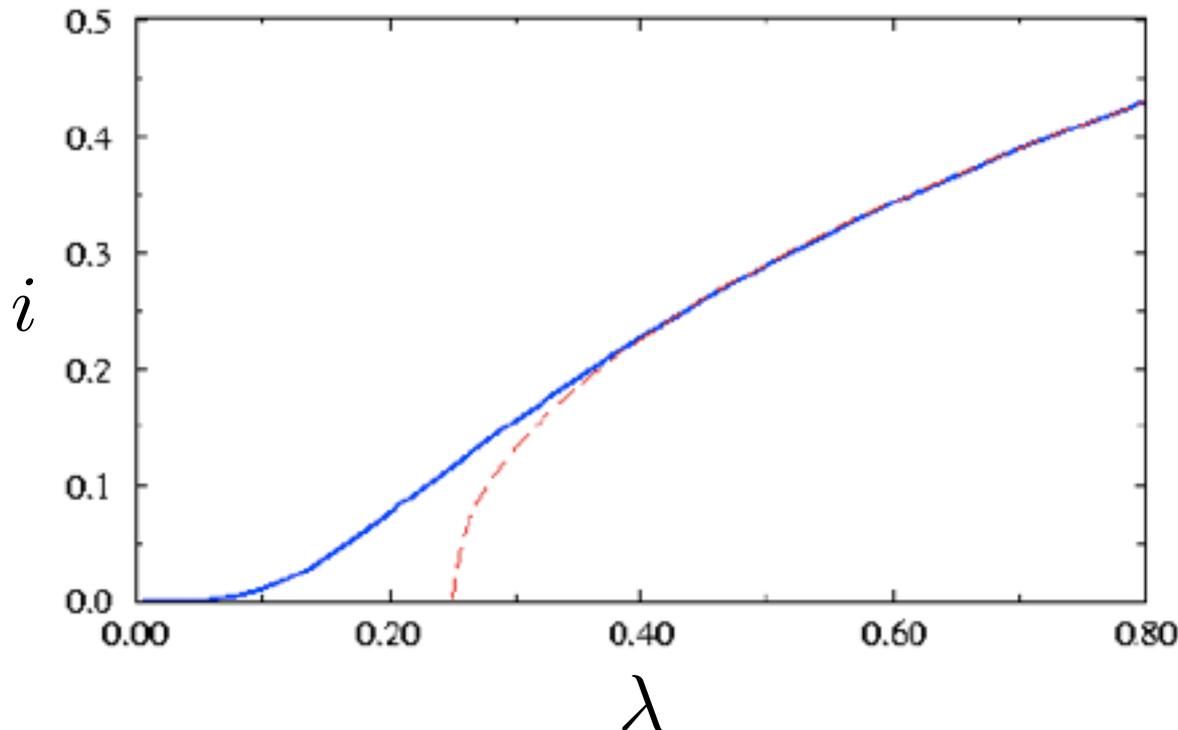
$$\lambda > \lambda_c = \frac{\mu \langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}$$

$$\langle k^2 \rangle \rightarrow \infty$$
$$\gamma \leq 3$$



$$\lambda_c \rightarrow 0$$

thermodynamic  
limit



the infection pervades  
the system whatever  
spreading rate

the healthy phase does  
not exist

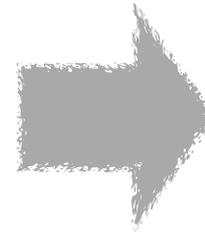
Cohen et al, PRL 2000  
Pastor-Satorras et al, PRL 2001  
Newman PRE 2002

# immunization in heterogeneous networks

epidemic is controlled if  $\lambda(1 - v) \leq \lambda_c$

but in infinite-size  
heterogeneous networks

$$\lambda_c = 0$$

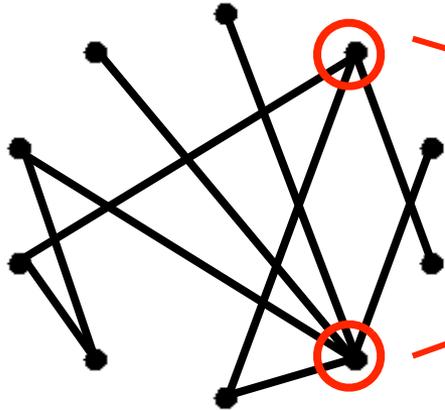


$$v_c = 1$$

immunization  
threshold !!!

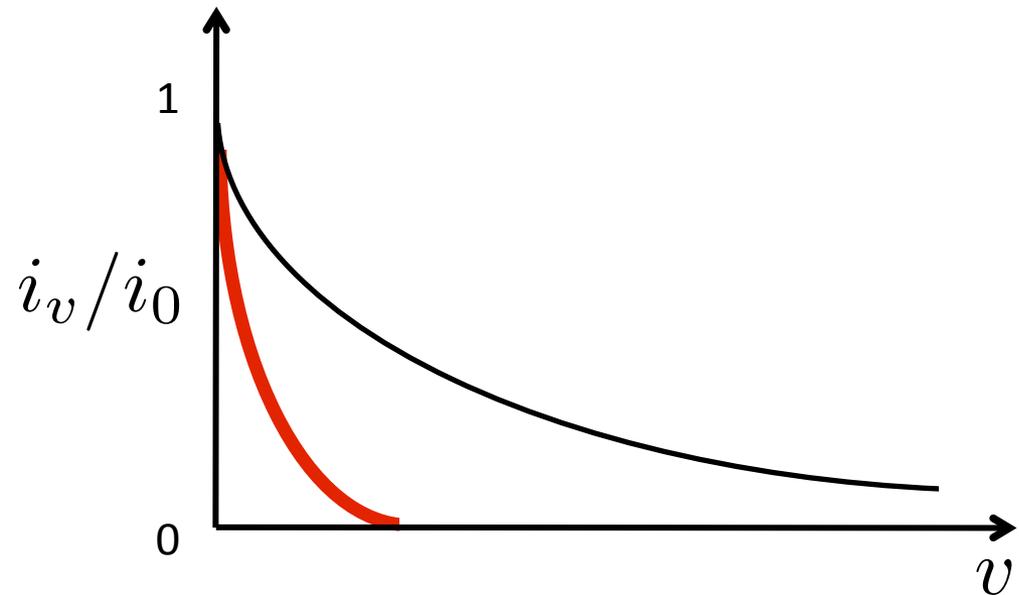
random immunization is *ineffective*... then what???

# targeted immunization strategy



progressive immunization  
of crucial nodes: **hubs**

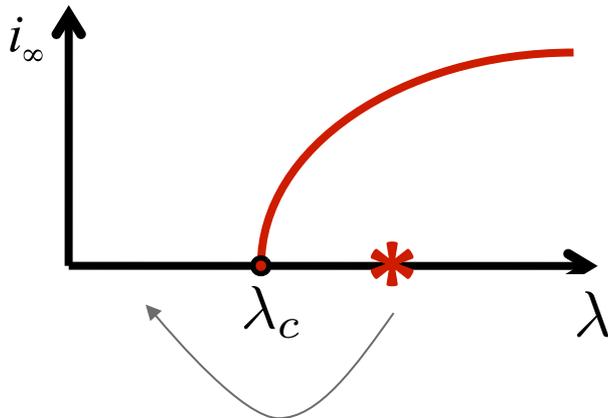
epidemic threshold is  
reintroduced



# SIR: homogeneous mixing vs. heterogeneous networks

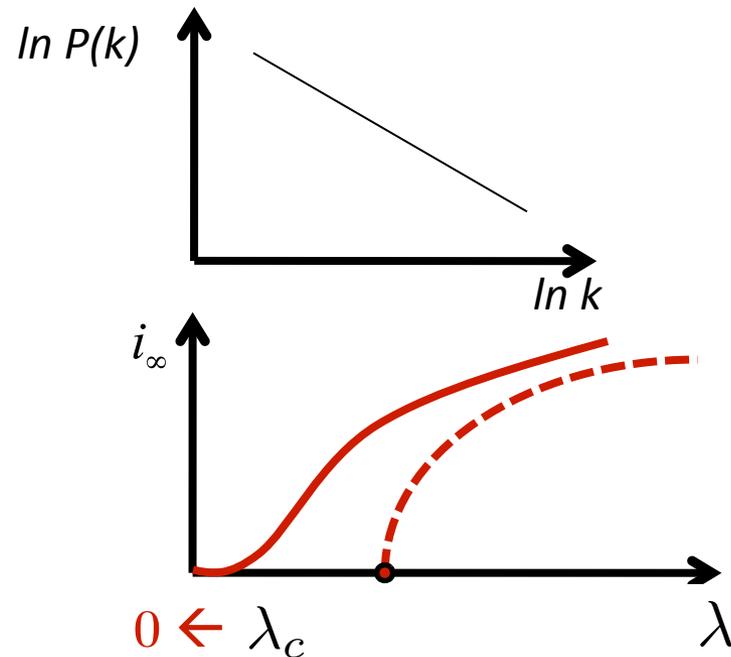
homogeneous mixing

$$k \simeq \langle k \rangle$$



epidemic threshold:  
active phase & healthy phase  
immunization threshold

heterogeneous contact pattern



vanishing epidemic threshold:  
healthy phase does not exist  
targeted immunization

# epidemic threshold for SIS

**see:** <http://arxiv.org/pdf/1010.1646.pdf>  
<http://arxiv.org/pdf/1105.5545.pdf>  
<http://arxiv.org/pdf/1206.6728.pdf>  
<http://arxiv.org/pdf/1305.4819.pdf>

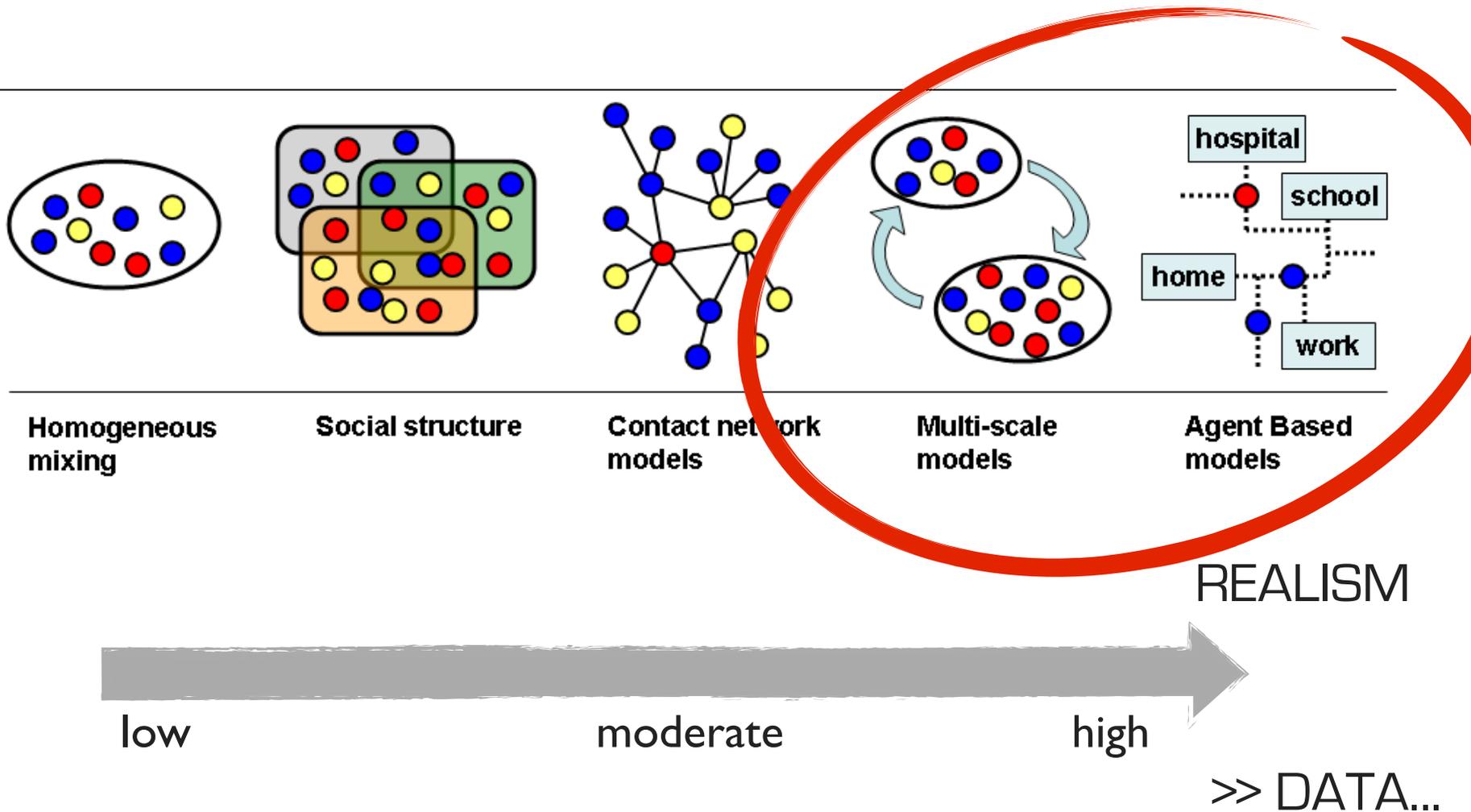
infectious diseases:  
space & metapopulation  
models

# population & space

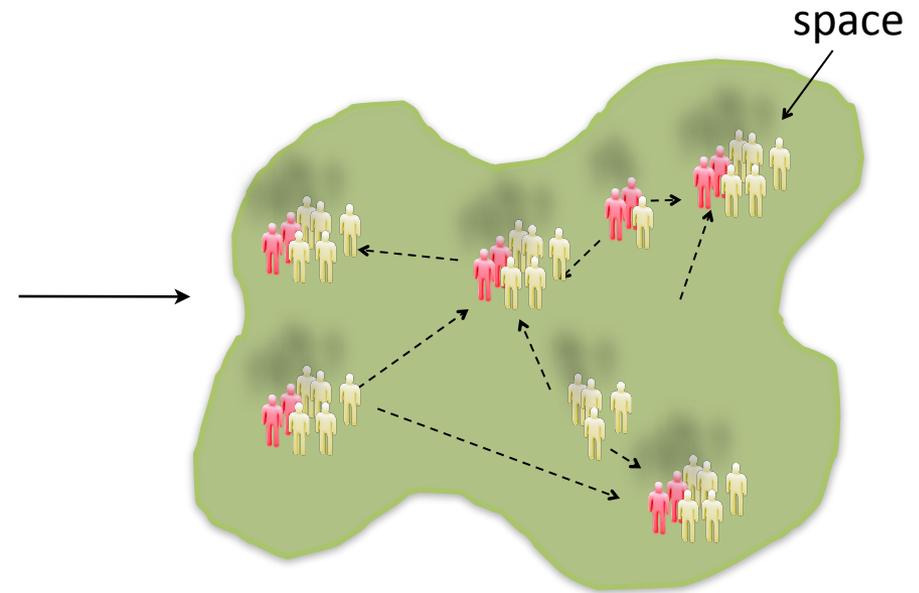
H1N1 influenza pandemic, 2009



# models' complexity



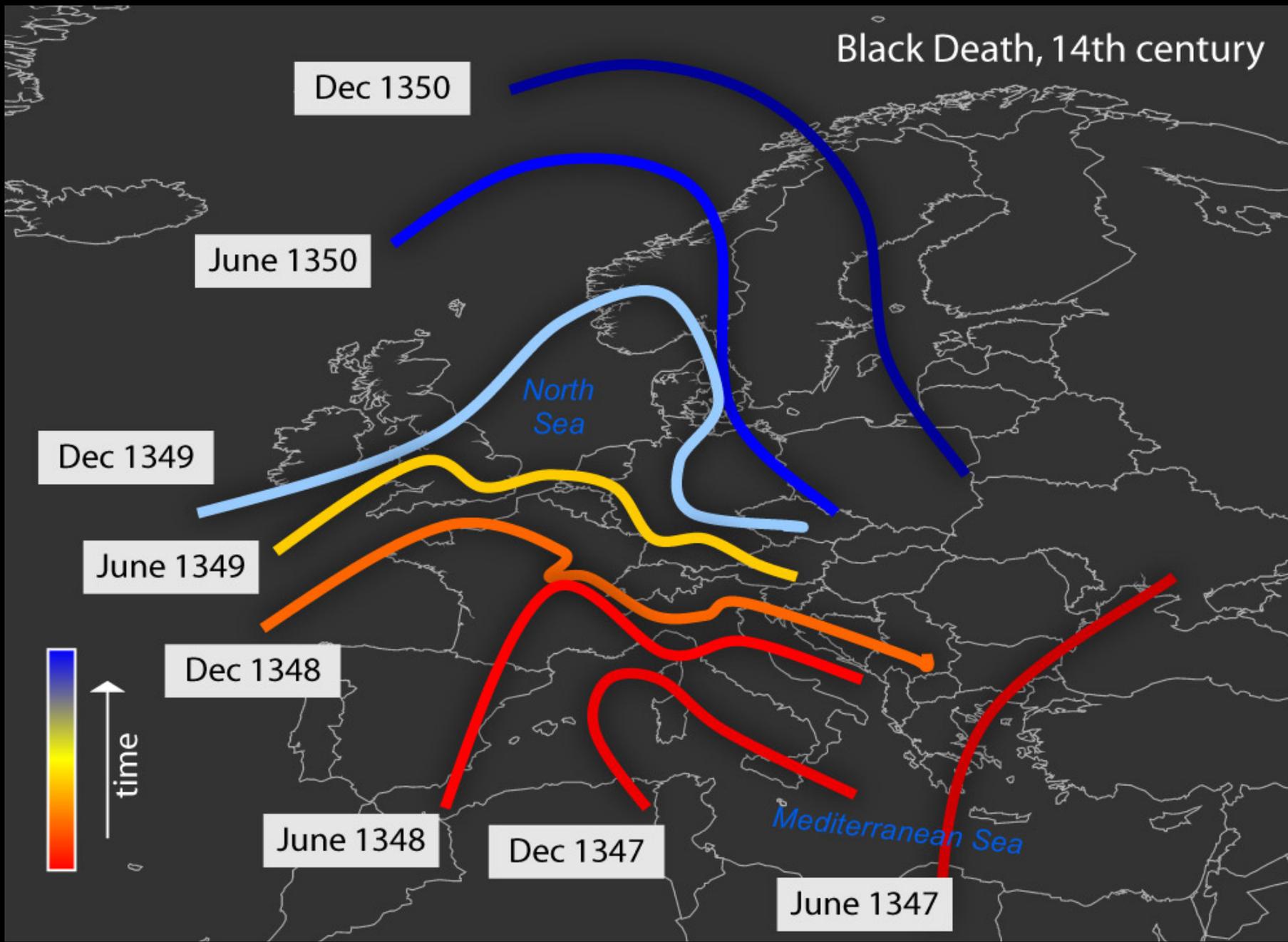
# modeling spatial spread



“Professional epidemiologists appear oblivious to where the epidemic is, asking only when numbers will appear along the time horizon.”

*geographer Peter Gould (1991)*

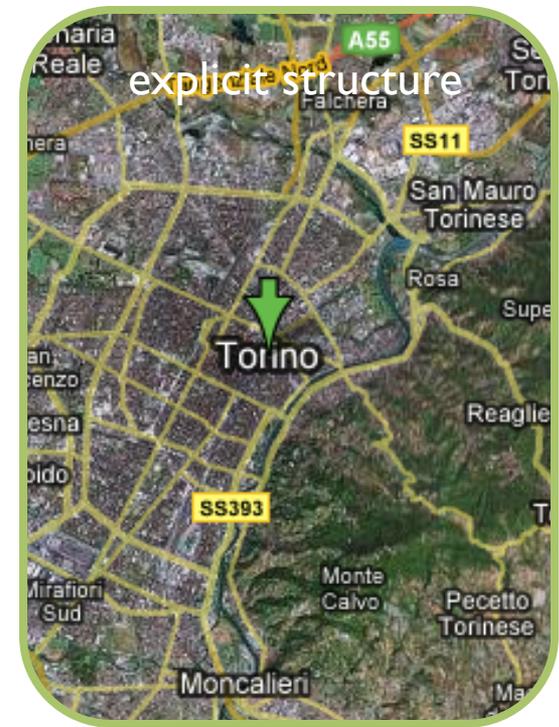
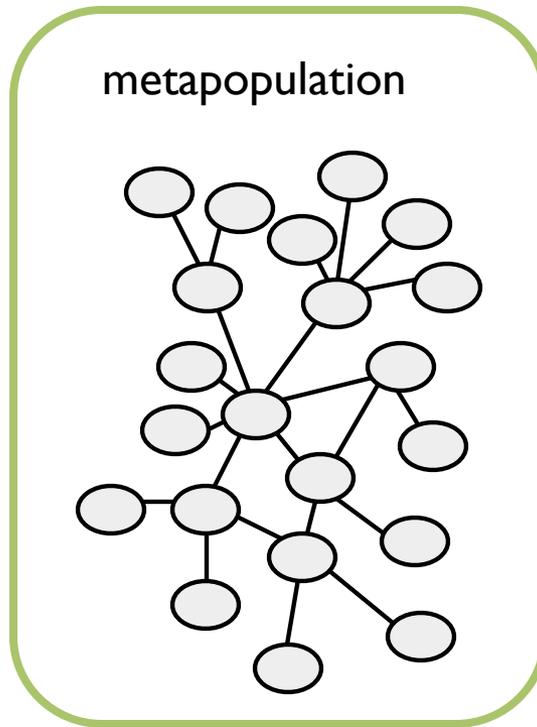
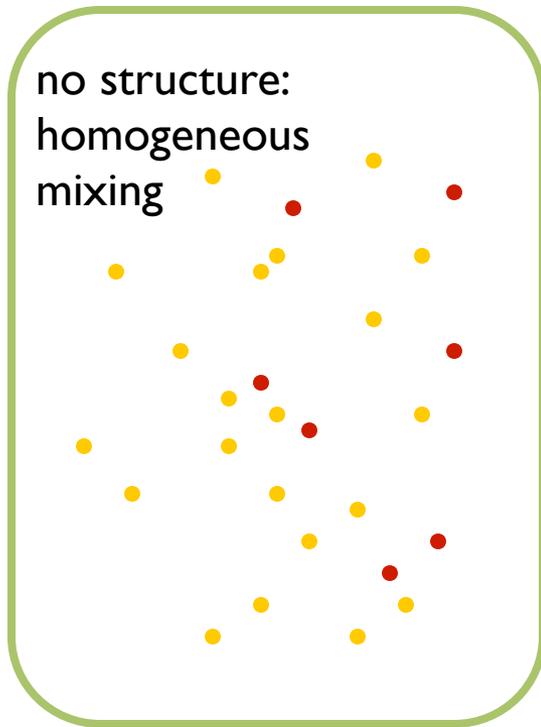
# Black Death, 14th century



# H1N1 influenza pandemic, 2009



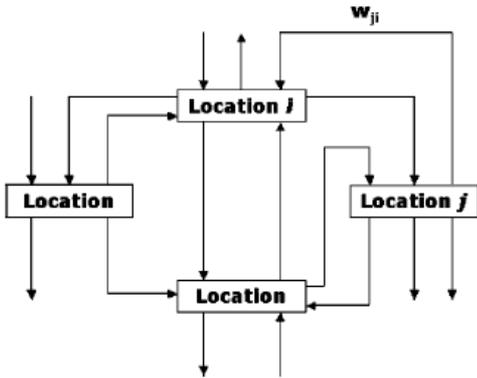
# metapopulation model: a compromise



local populations:

- discrete entities in space: *patches*
- interaction between populations : *coupling, flows*

# patches & coupling



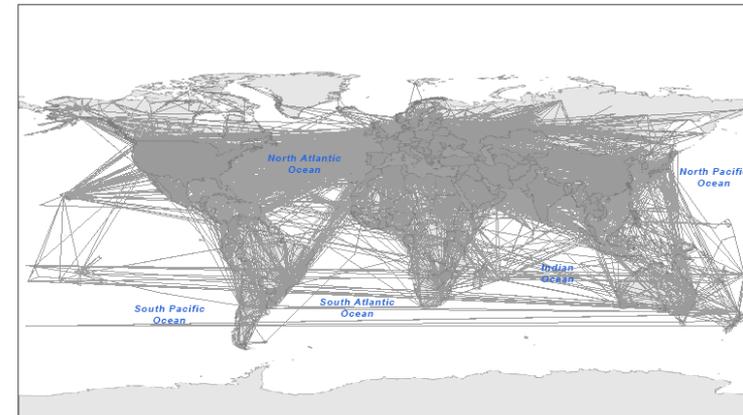
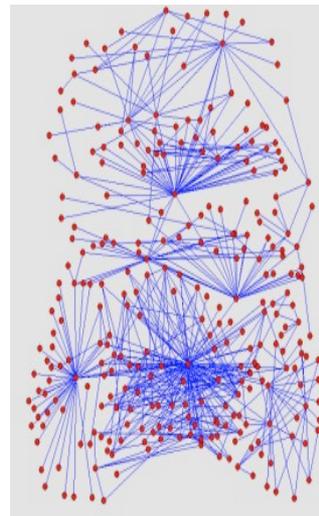
movement of people  
between in-city locations

commuting patterns

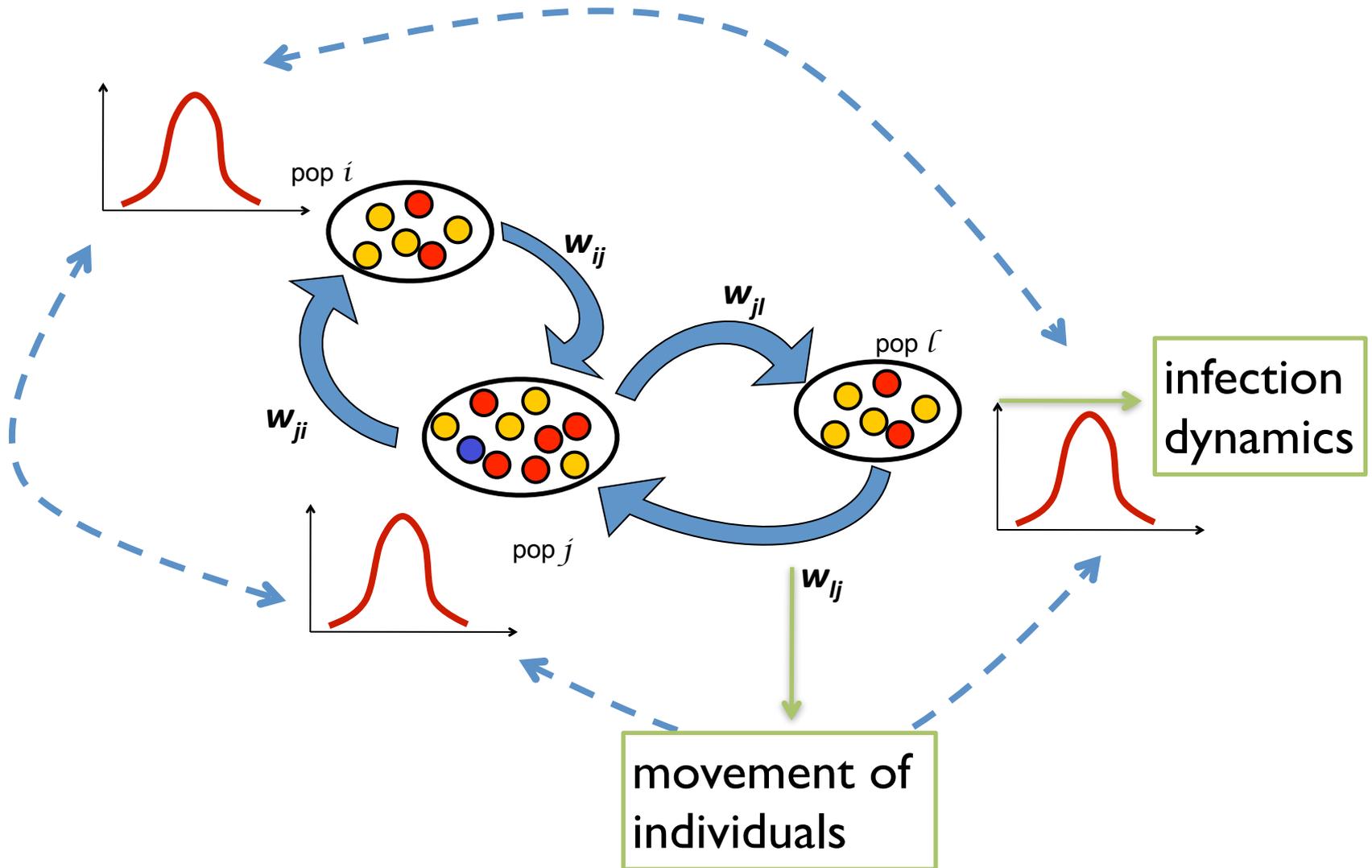
air travel

TABLE I. Sample section of a TRANSIMS activity file. In this example, person 115 arrives for a social recreational activity at location 33 005 at 19.25 o'clock and departs at 21.00 o'clock.

Person ID	Location ID	Arrival time	Departure time	Activity type
115	4225	0.0000	7.00	Home
115	49 296	8.00	11.00	Work
115	21 677	11.2	13.00	Work
115	49 296	13.2	17.00	Work
115	4225	18.00	19.00	Home
115	33 005	19.25	21.00	Social/rec
115	4225	21.3	7.00	Home
220	8200	0.0000	8.50	Home
220	10 917	9.00	14.00	School
220	8200	14.5	18.00	Home
220	3480	18.2	20.00	Social/rec
220	8200	20.3	8.6	Home



# metapopulation models



# SIR metapopulation models

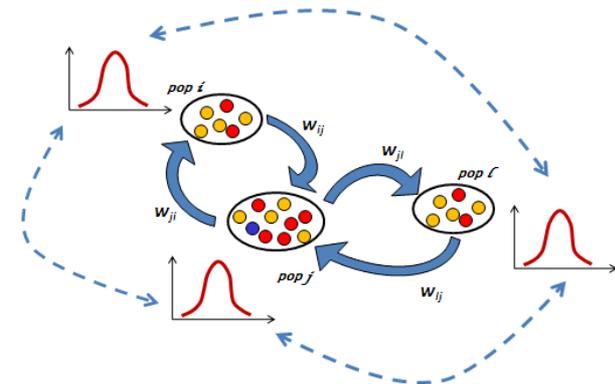
$$\frac{dS_i}{dt} = -\beta \frac{I_i(t)S_i(t)}{N_i} + \Omega_i^S$$

$$\frac{dI_i}{dt} = \beta \frac{I_i(t)S_i(t)}{N_i} - \mu I_i(t) + \Omega_i^I$$

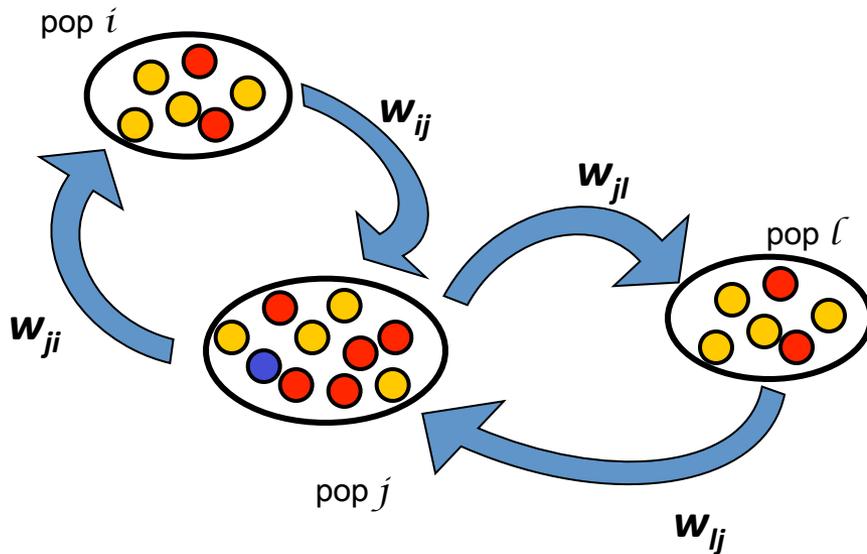
$$\frac{dR_i}{dt} = \mu I_i(t) + \Omega_i^R$$

$\Omega_i^X$

Measure of *in-flow* and *out-flow* of people in compartment  $X$



# coupling



- ▶ for all compartments
- ▶ for a subset of compartments

probability to travel  
from  $i$  to  $j$ ?

$$p_{ij} = p$$

$$p_{ij} = \frac{w_{ij}}{\sum_j w_{ij}}$$

$$p_{ij} = \frac{w_{ij}}{N_i}$$