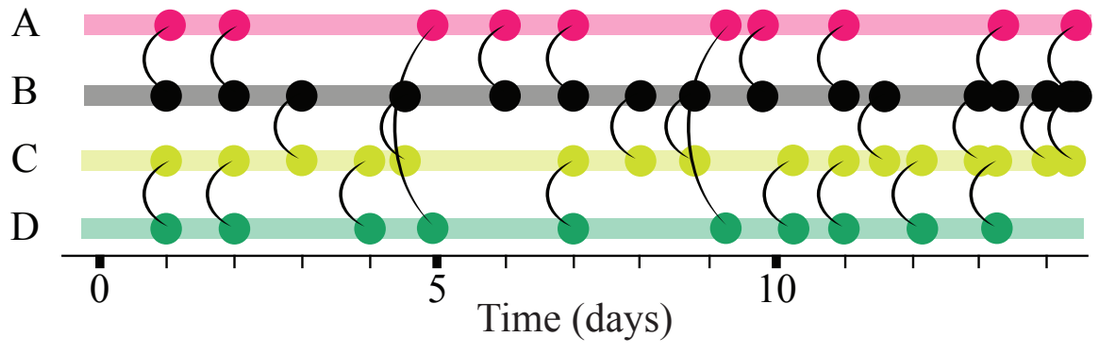
Temporal Networks

3. Burstiness & temporal inhomogeneities



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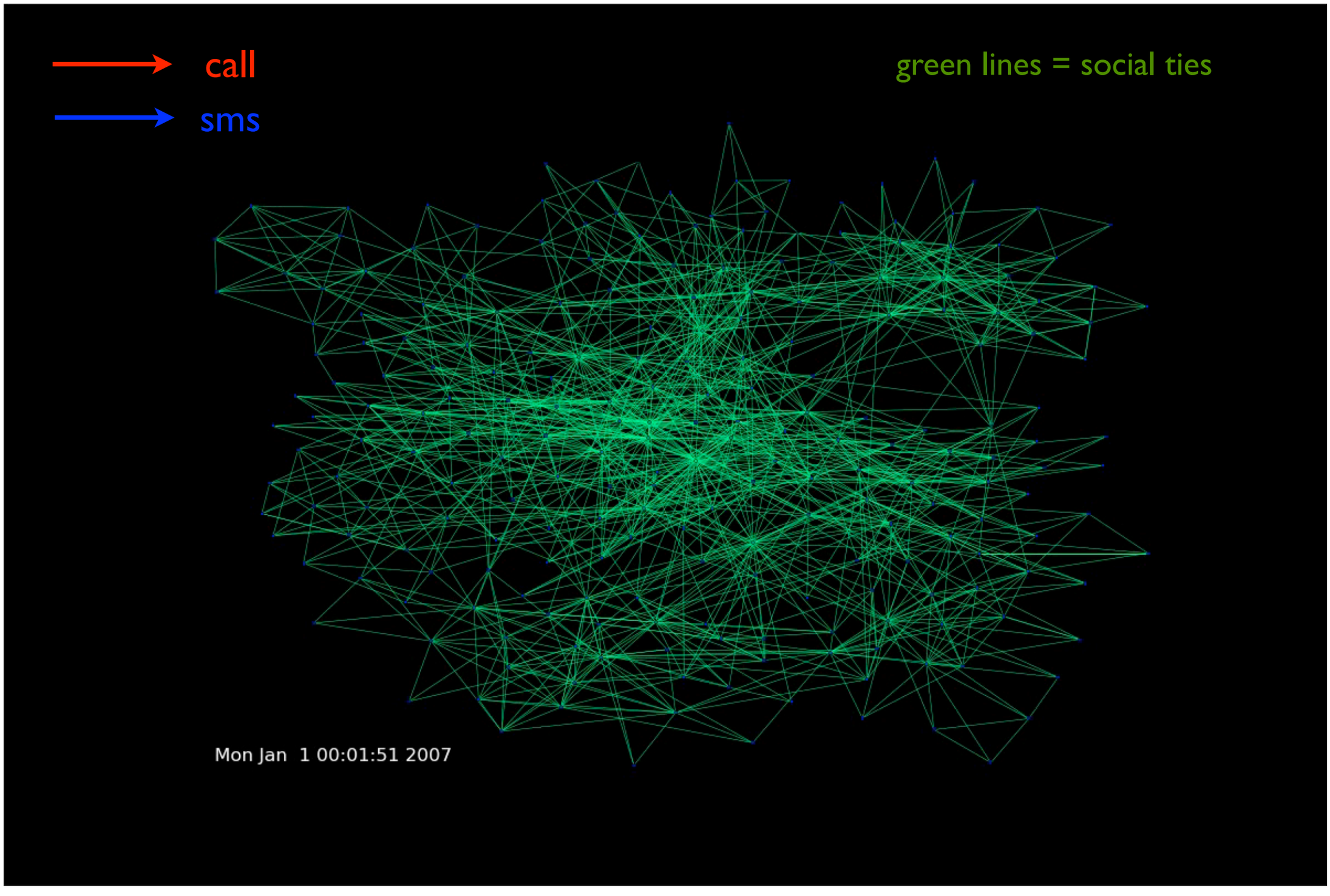
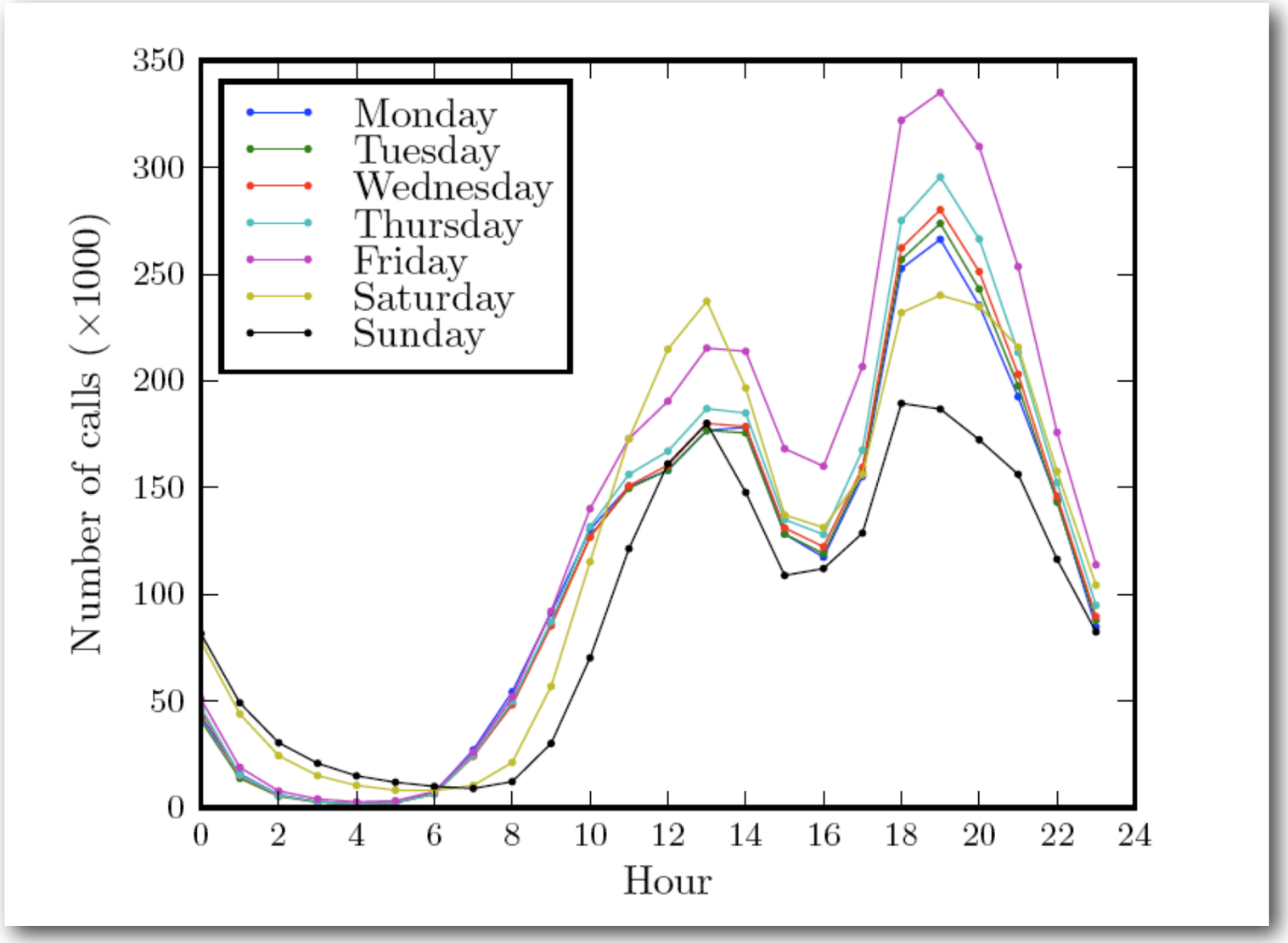
- static network structure far from homogeneous (e.g. that of Erdös-Rényi networks):
  - broad degree distributions
  - mesoscopic structure

- what about temporal structure?


call
sms
green lines = social ties
Mon Jan 1 00:01:51 2007

Network-level timing correlations



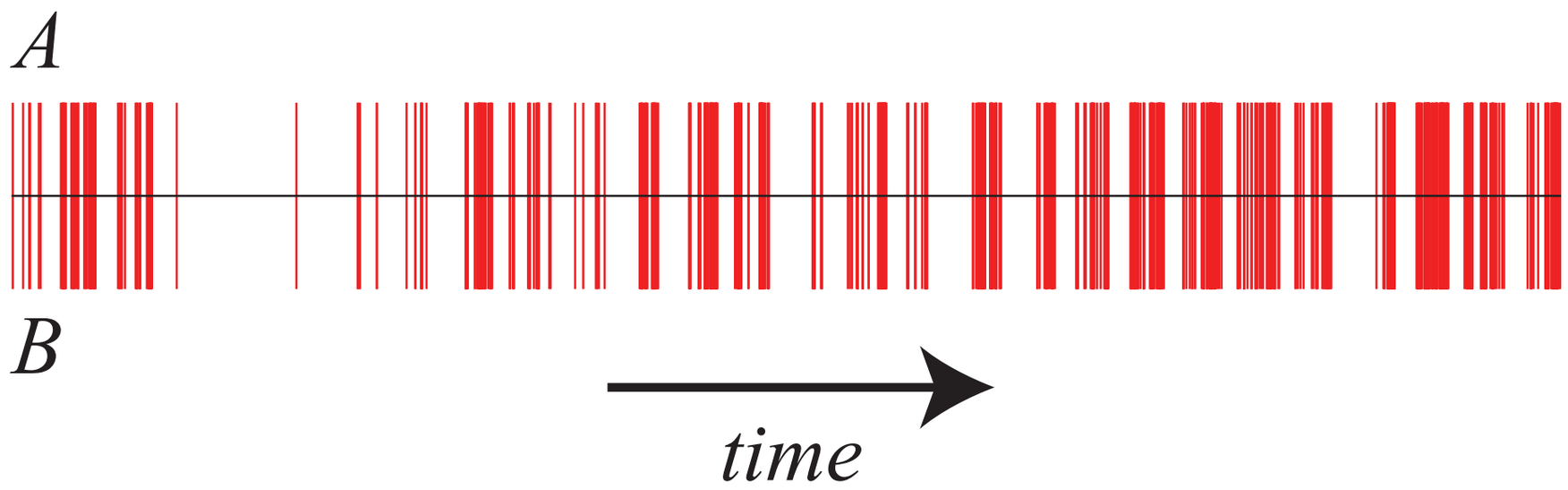
human communication dynamics: daily & weekly patterns

Temporal inhomogeneities: inter-event times

a sequence of phone calls between A & B

there is *burstiness* in the sequence

Temporal inhomogeneities: inter-event times

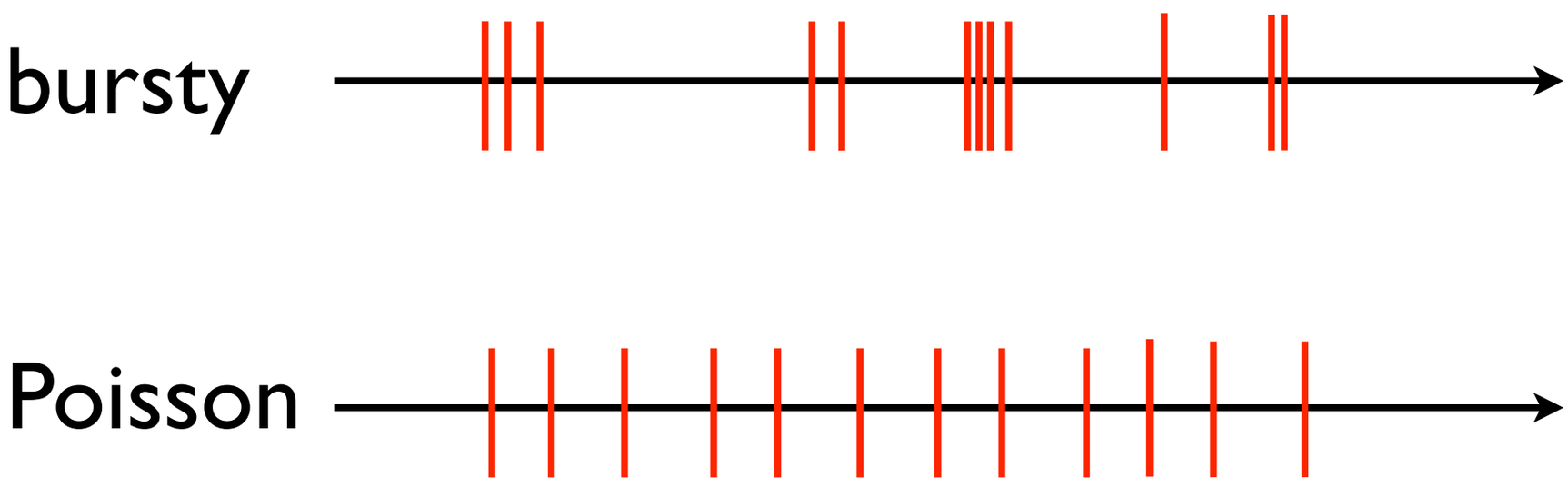


a sequence of phone calls between A & B

there is *burstiness* in the sequence

Burstiness

- burstiness: **events clustered in time**
- reference: a homogeneous Poisson process with constant event probability per unit time



Measuring burstiness

$$B = \frac{(\sigma_\tau / m_\tau - 1)}{(\sigma_\tau / m_\tau + 1)} = \frac{(\sigma_\tau - m_\tau)}{(\sigma_\tau + m_\tau)}.$$

m_τ = mean inter-event time

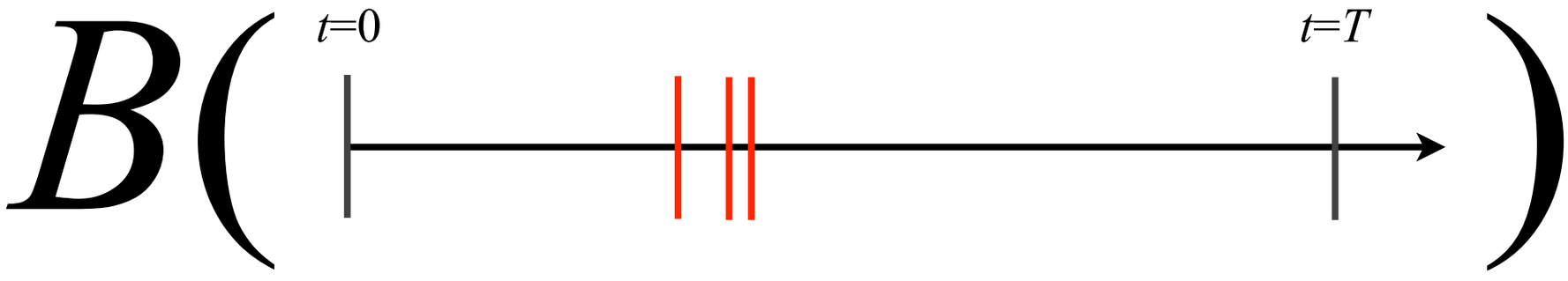
σ_τ = std dev of inter-event times

$B = -1$	$B = 0$	$B = 1$
periodic	Poisson	max. bursty

this quantity can be calculated for nodes or links

K.-L. Goh, A.-L. Barabási Burstiness and memory in complex systems. Europhysics Letters 81, 48002 (2008)

Problems with B



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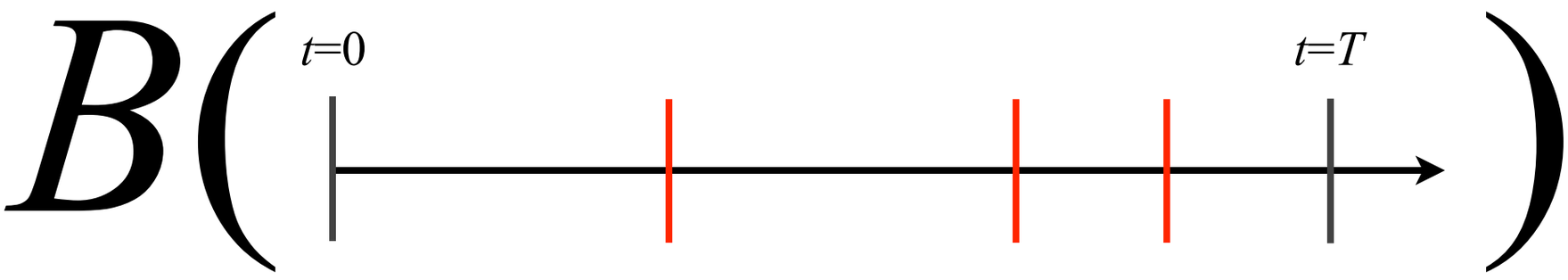
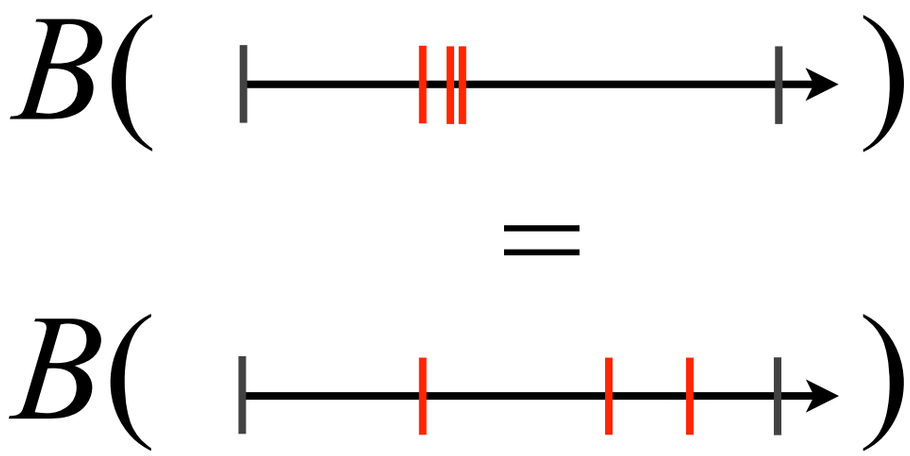


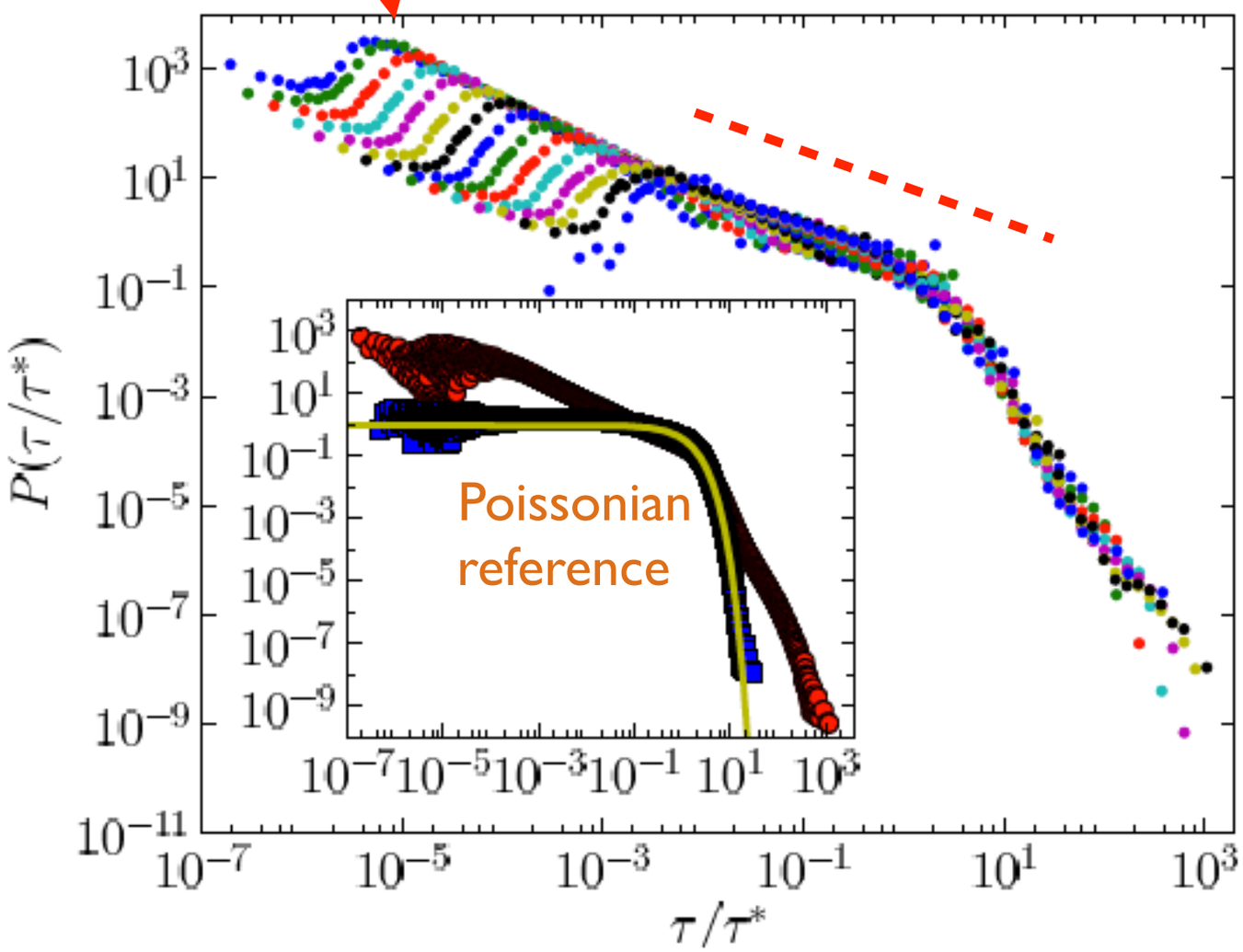
Problems with B - solutions?



- use periodic boundary conditions?
- add "virtual events" at $t=0$, $t=T$?
- in any case, low-activity links/nodes difficult.

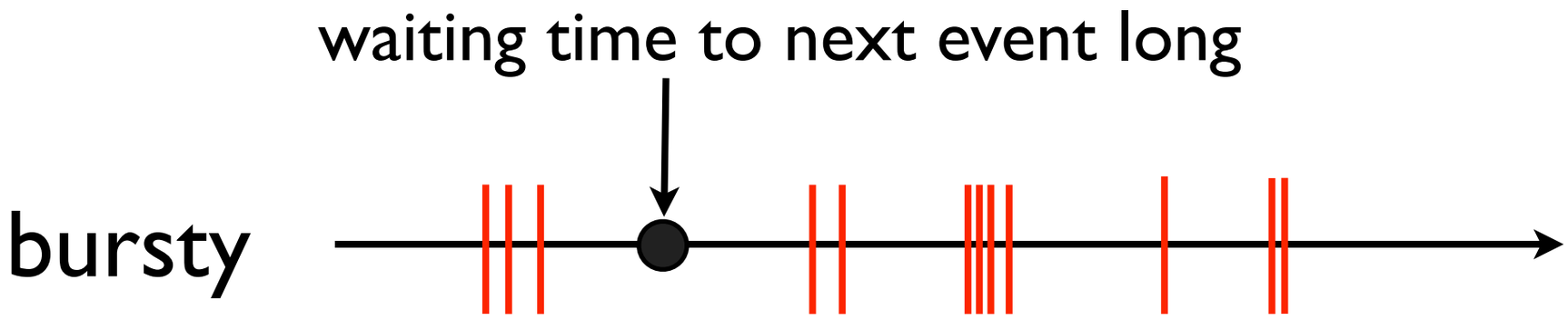
Burstiness: broad inter-event time distributions

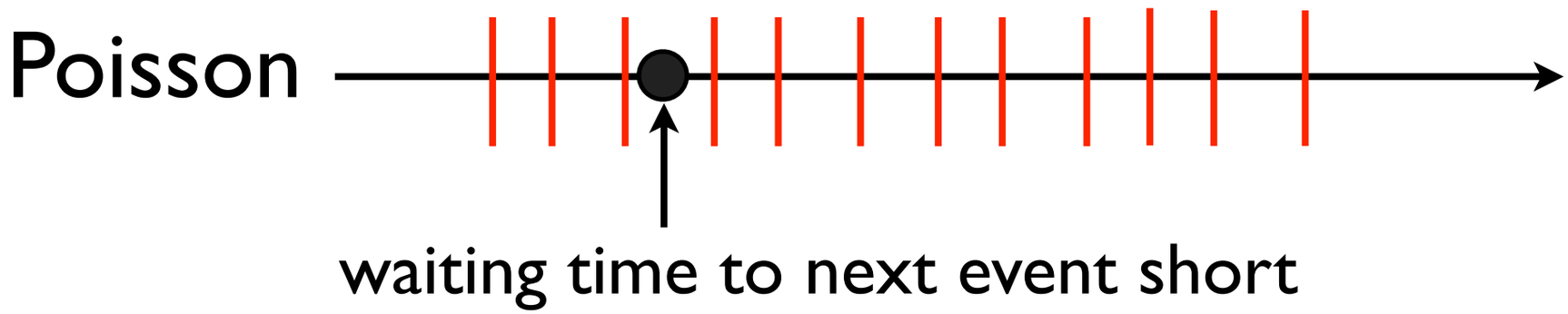
this non-scaling part is because of other correlations - we'll get there soon...



- Events = phone calls
- Nodes binned according to # of calls of node
- Inter-event time distribution scaled by average inter-event time in bin

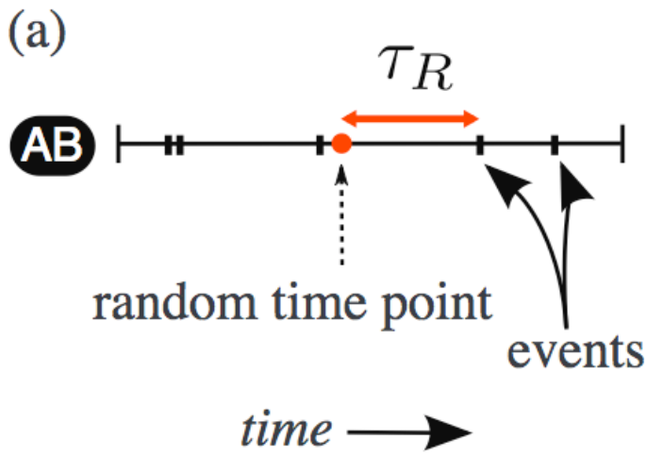
Consequences of burstiness





generally, (uncorrelated) burstiness
increases latency of temporal paths

Relay time & burstiness



$$\overline{\tau_R}^* \equiv \frac{\overline{\tau_R}}{\overline{\tau_{R,p}}} = \frac{\overline{\tau^2}}{\overline{\tau_p^2}} = \frac{\overline{\tau^2}}{\overline{\tau}^2}$$

- Avg. relay time $\overline{\tau_R}$ = waiting time until next event from a random time point
- Normalized relay time $\overline{\tau_R}^*$ = divided by the relay time of a Poisson process with same mean inter-event time
- $\overline{\tau_R}^* = \frac{B^2+1}{(B-1)^2}$

M. Kivelä et al., J. Stat. Mech. P03005 (2012)

Temporal reference models

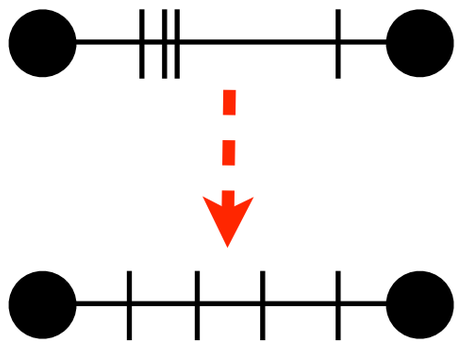
Idea: remove correlations, observe effects on dynamics

- static networks:
  all correlations topological $\Rightarrow$ configuration model
- temporal networks:
  many types of correlations $\Rightarrow$ many reference models

Temporal reference models

Random times

- For each link's each event, pick the time at random
- Destroys all temporal correlations



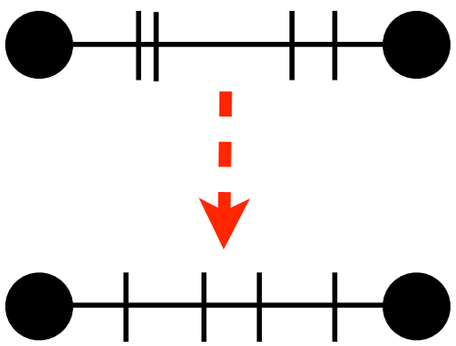
(optional: randomize between first and last event of link only)

Time-shuffling

- Randomly shuffle time stamps of all calls
- Destroys all temporal correlations except for network-level patterns (e.g. daily patterns)

caller	callee	time
A	B	1
D	E	3
D	F	9
F	A	11

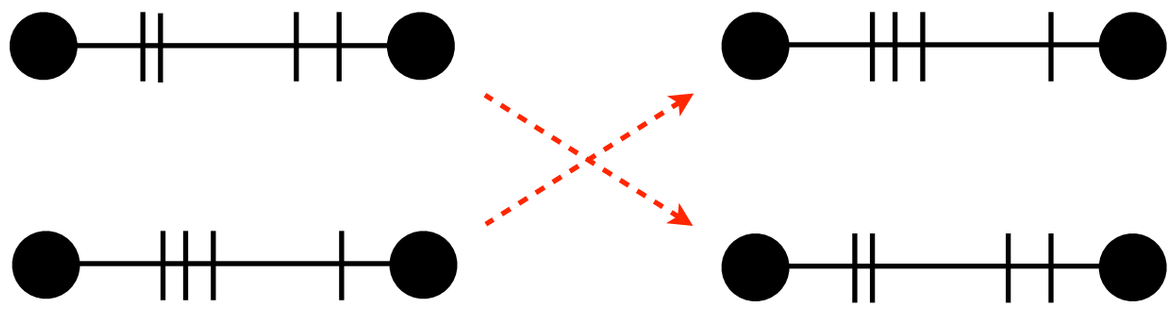
caller	callee	time
A	B	9
D	E	11
D	F	3
F	A	1



Temporal reference models

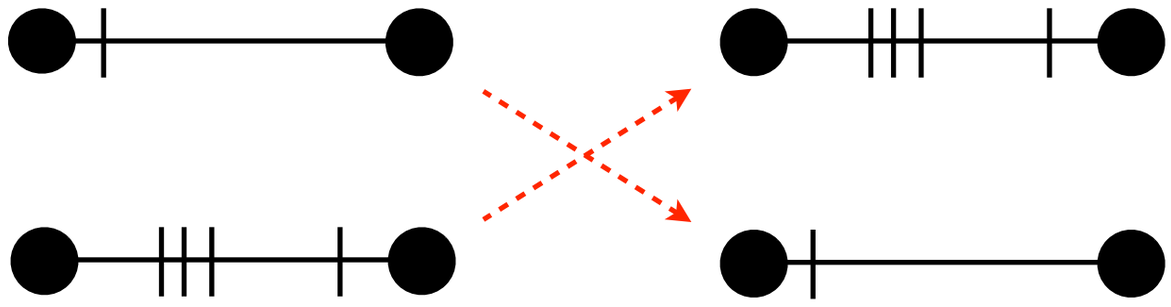
Weight-conserving link-sequence exchange

- Randomly exchange entire event sequences
- Removes correlations btw adjacent links, retains sequence characteristics & link weights



Link-sequence exchange

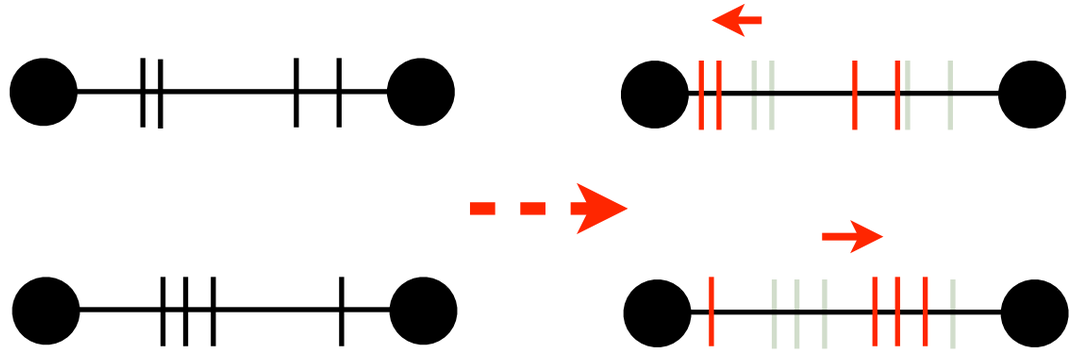
- Randomly exchange entire event sequences



Temporal reference models

Random link-sequence shift

- Shift events on links by some random amount (use periodic temporal boundary conditions)
- Removes correlations btw adjacent links, retains exact link sequences

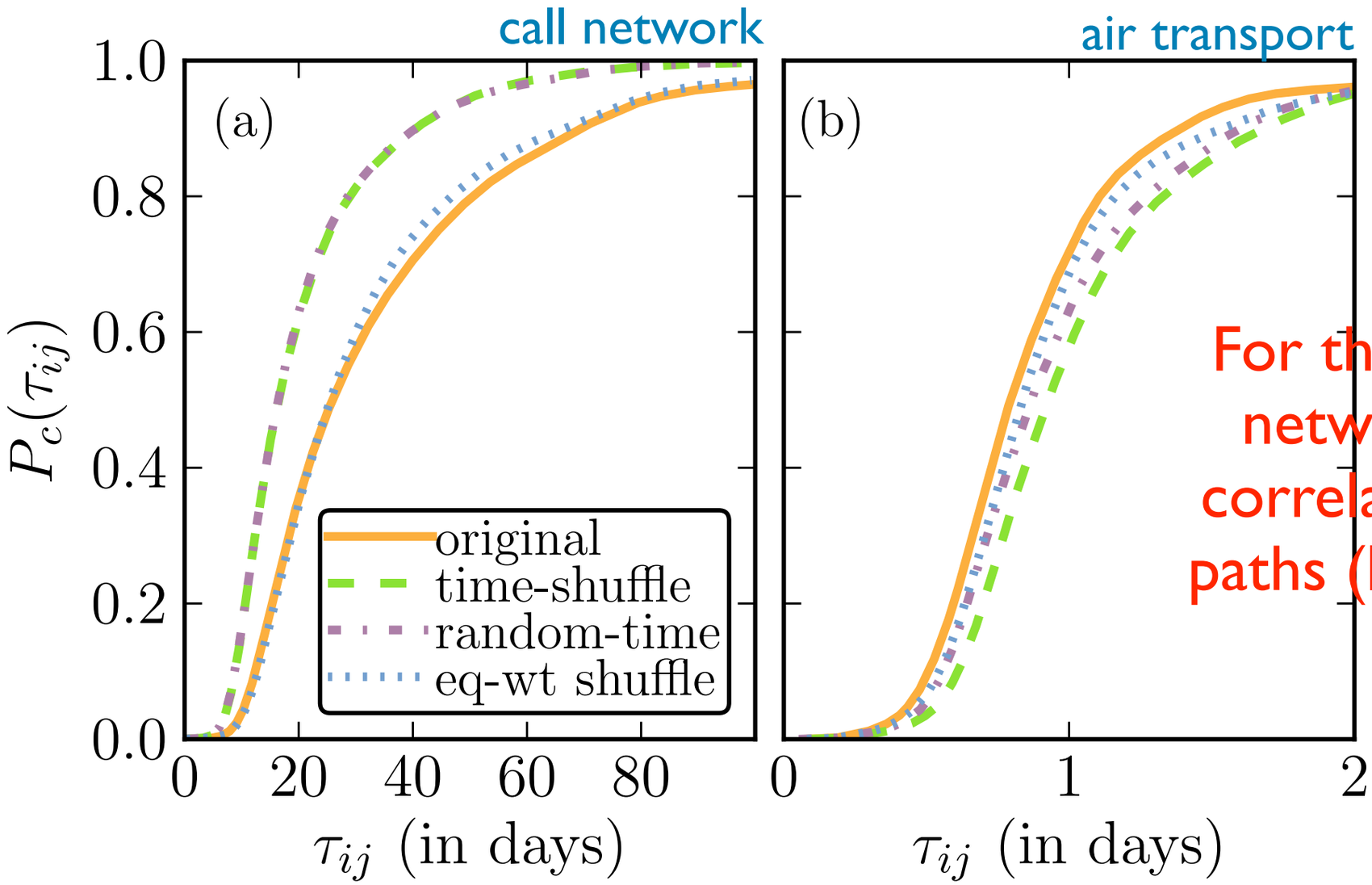


Inter-event time shuffle on links

- Randomize sequence of inter-event times
- Retains burstiness of links; destroys any other correlations within links & between links

Latencies & temporal correlations

latency distribution (all paths), original & reference models



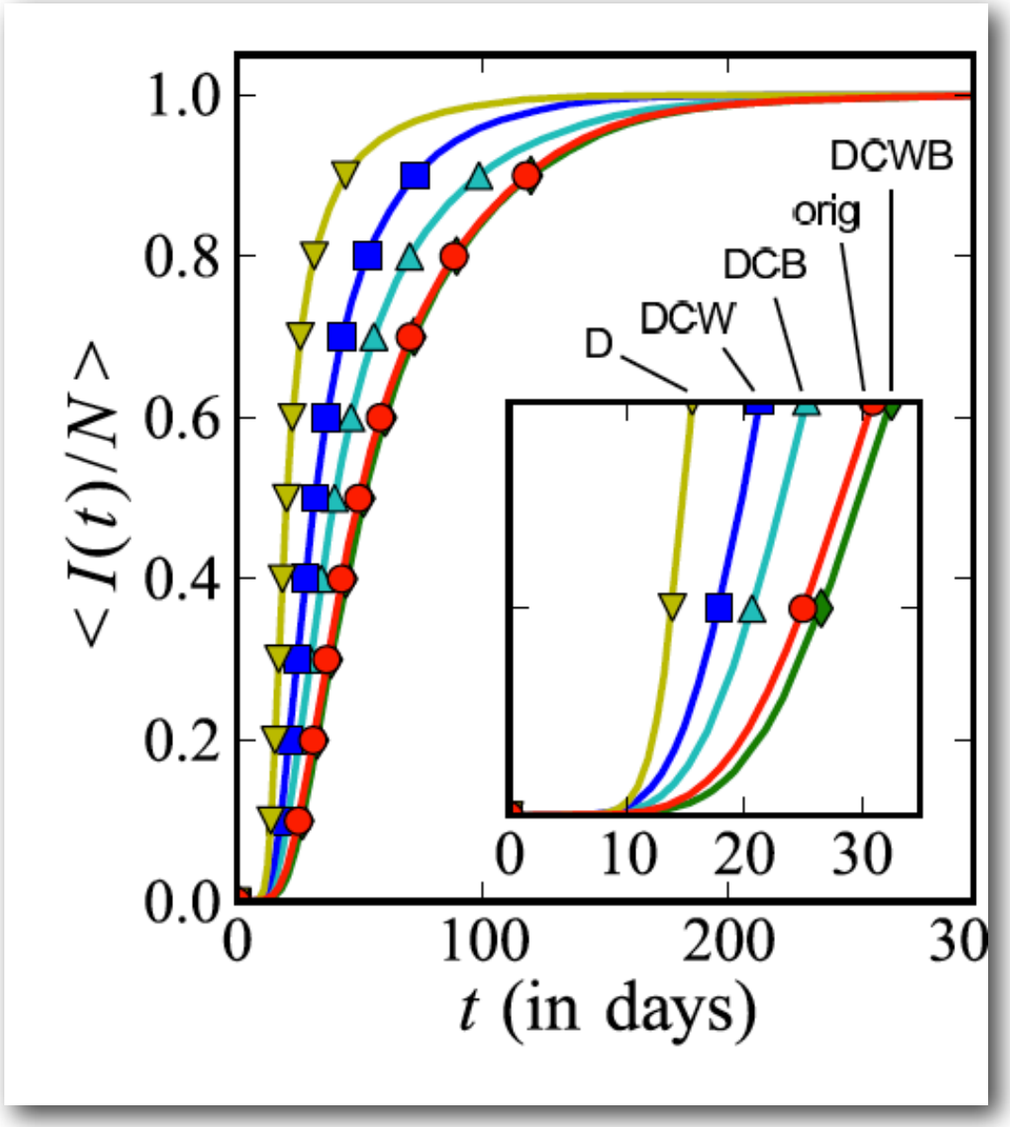
For the air transport network, temporal correlations speed up paths (lower latencies)

Path lengths, correlations, and centrality in temporal networks, R.K. Pan and J. Saramäki, Phys. Rev. E 84, 016105 (2011)

SI spreading dynamics

- Proceeds along fastest paths
- Nodes either Infected (I) or Susceptible (S)
- Initialize: nodes S, except for seed I node
- At every contact between I & S nodes, S becomes I.

prevalence vs time
- curves for null models



Small but slow world: how network topology and burstiness slow down spreading, M. Karsai, M. Kivelä, R. K. Pan, K. Kaski, J. Kertész, A.-L. Barabási, and J. Saramäki, Phys Rev E **83**, 025102(R) (2011)

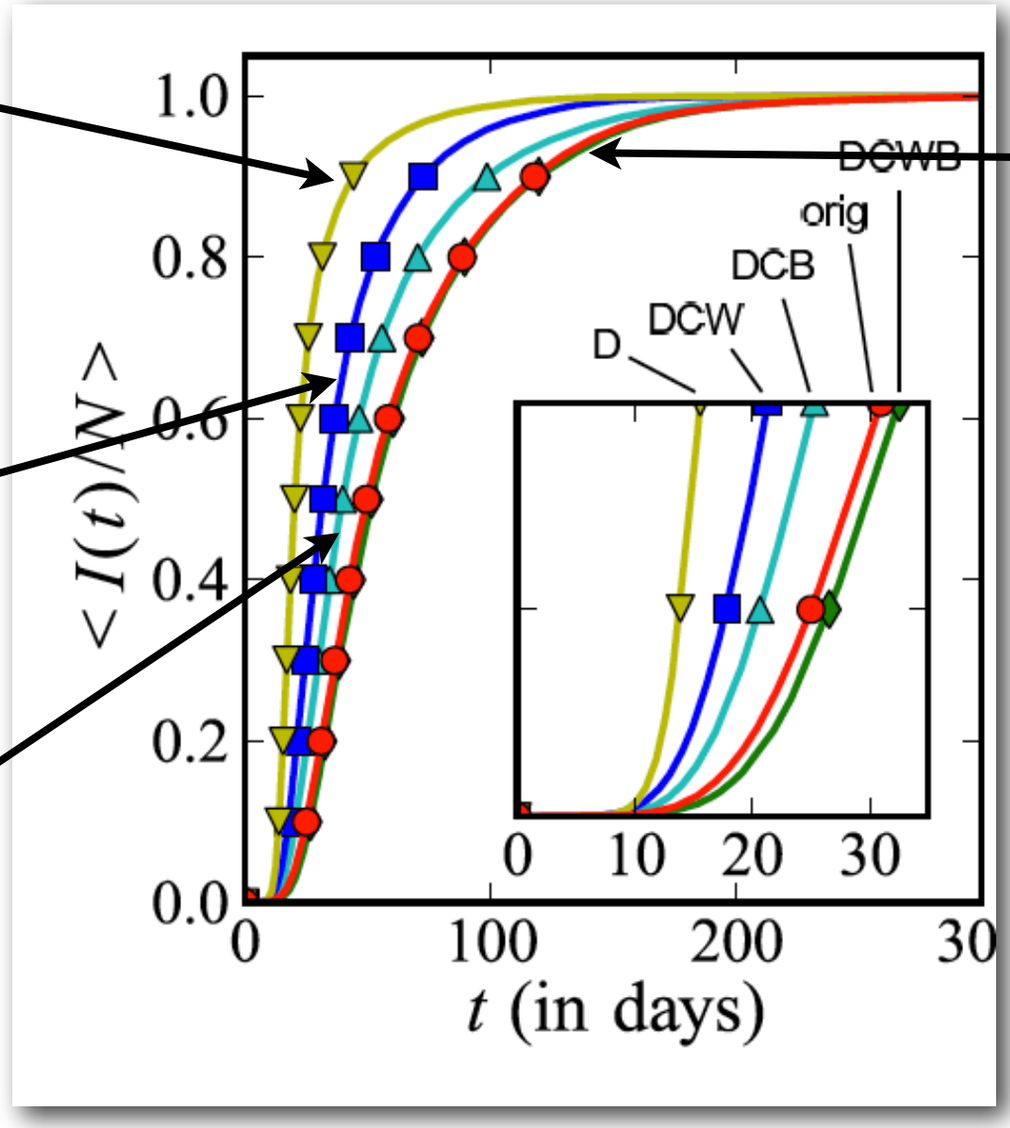
See also: A. Vazquez, B. Rácz, A. Lukács and A.-L. Barabási, PRL **98**, 158702 (2007); J.L. Iribarren and E. Moro, PRL **103**, 038702 (2009)

prevalence vs time
- curves for null models

most correlations removed

link burstiness removed

burstiness on, weak-link bottleneck removed



original call sequence

Bursty dynamics slows down spreading a lot!

Small but slow world: how network topology and burstiness slow down spreading, M. Karsai, M. Kivelä, R. K. Pan, K. Kaski, J. Kertész, A.-L. Barabási, and J. Saramäki, Phys Rev E **83**, 025102(R) (2011)

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Burstiness is (fairly) universal

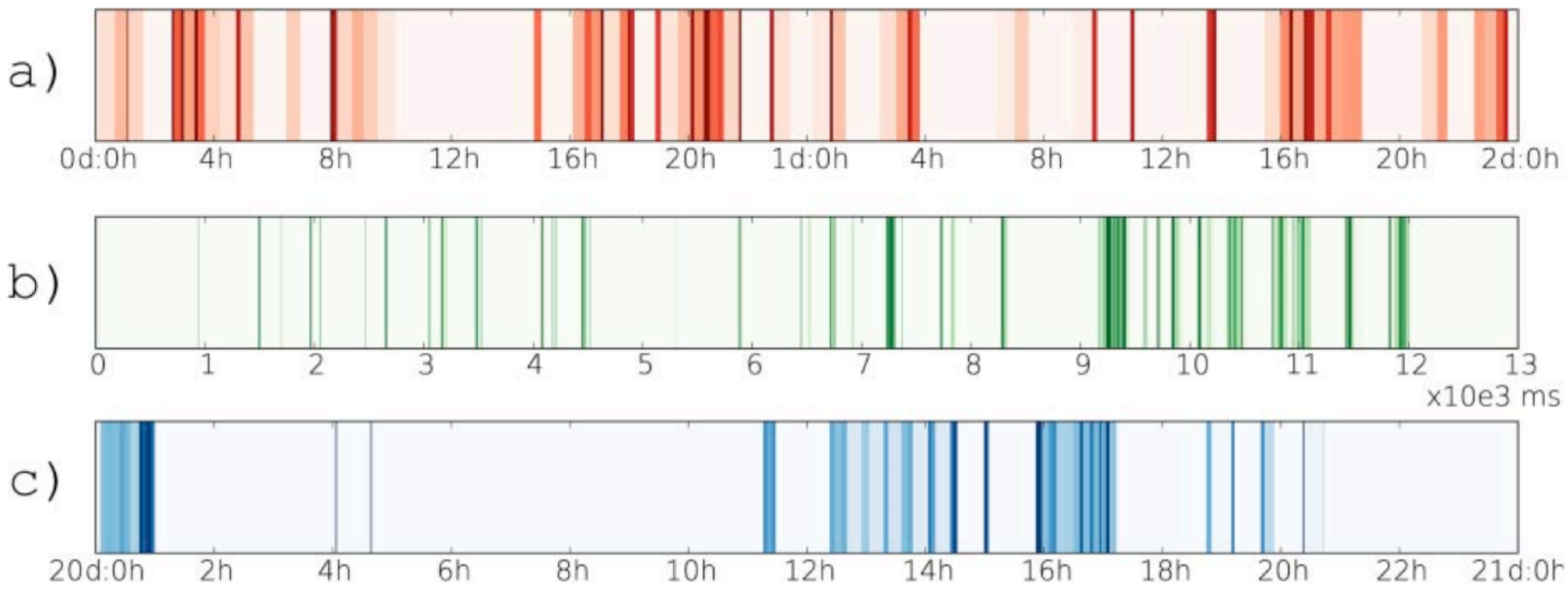


Figure 1 | Activity of single entities with color-coded inter-event times. (a): Sequence of earthquakes with magnitude larger than two at a single location (South of Chishima Island, 8th–9th October 1994) (b): Firing sequence of a single neuron (from rat's hippocampal) (c): Outgoing mobile phone call sequence of an individual. Shorter the time between the consecutive events darker the color.

from Karsai et al, Scientific Reports **2**, 397 (2012)

Barabási priority queue model

a) with probability p, pick the task with highest priority, with probability $(1\text{-}p)$, pick a random task

b) execute the task

c) generate new task with random priority

d) sort tasks, repeat

task	priority
1	x_1
2	x_2
3	x_3
4	x_4
5	x_5
6	x_6
...	...
L	x_L

A.-L. Barabási, Nature (2005)

Correlations in bursty sequences

Question: are there correlations "hidden" within bursty event sequences?

Reference: renewal process (no correlations) with power-law inter-event times:

inter-event time distribution $P(\tau) \propto \tau^{-\beta}$

autocorrelation function $\mathcal{A}(\tau) \propto \tau^{-\alpha}$ ← $\mathscr{A}(t) = \frac{\mathbb{E}[X(0)X(t)] - \langle \mathbb{E}[X(t)] \rangle_t^2}{\langle \mathbb{E}[X(t)] \rangle_t - \langle \mathbb{E}[X(t)] \rangle_t^2}$

relation between exponents $\alpha + \beta = 2$

Correlations in bursty sequences

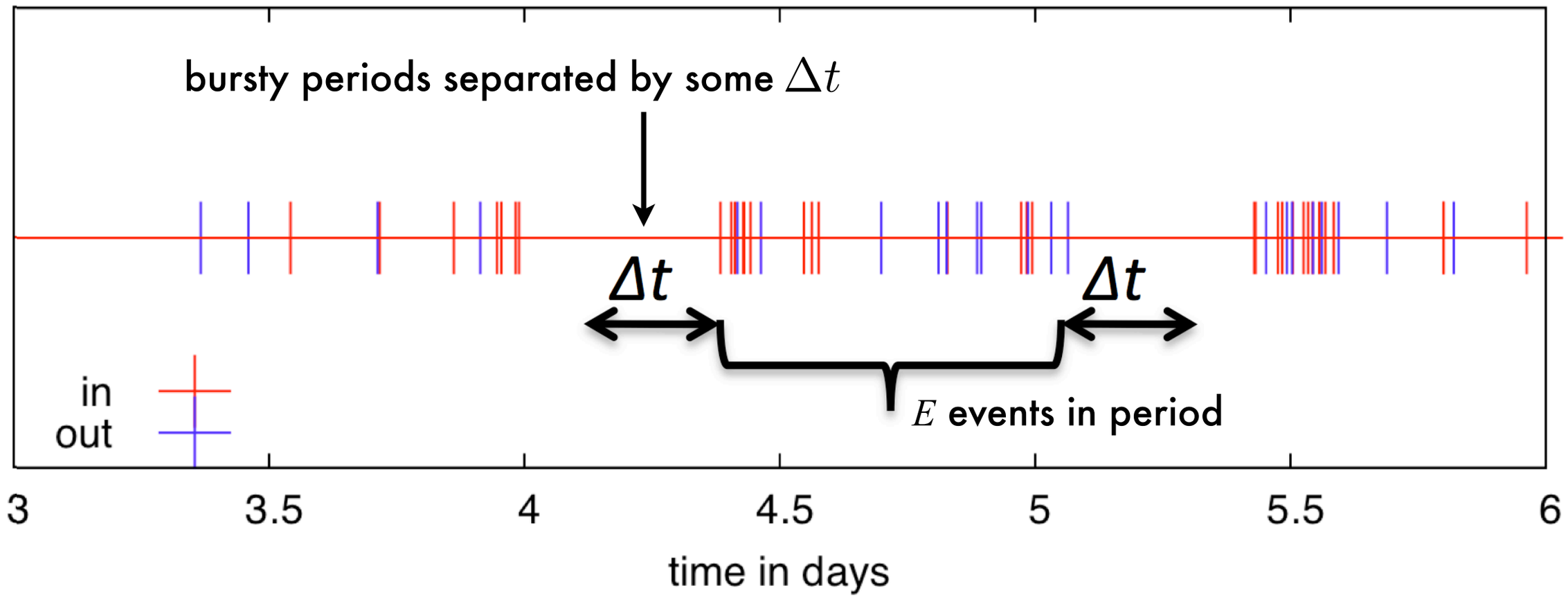
empirical data

	α	β	$\alpha+\beta$
Phone calls	0.5	0.7	1.2
SMS	0.6	0.7	1.3
Emails	0.7	1.0	1.7

$$\alpha + \beta < 2!$$

Karsai et al, Scientific Reports **2**, 397 (2012)

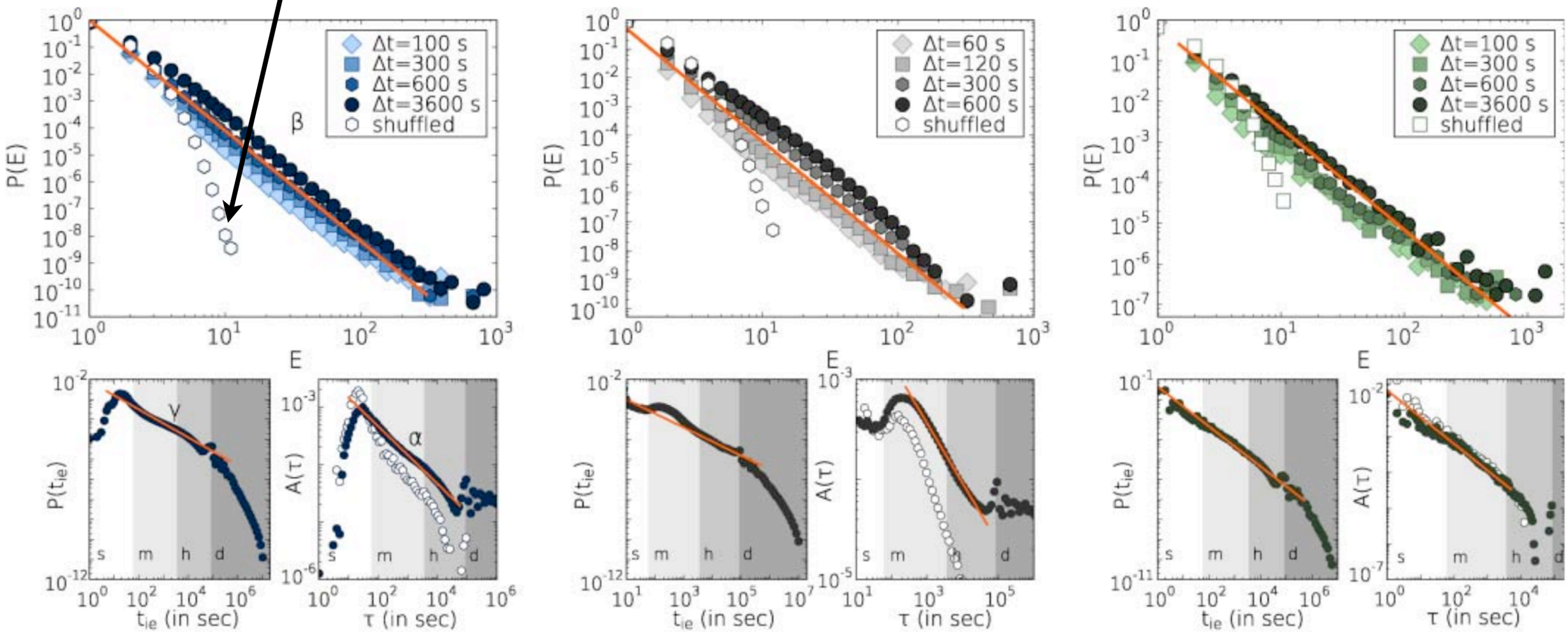
Bursty periods: definition



- Split event trains to periods
- Count the number of events in each
- Calculate the PDF, $P(E)$

Karsai et al, Scientific Reports **2**, 397 (2012)

shuffling: for each link, randomize the order of inter-event times



(a) Mobile-call sequence

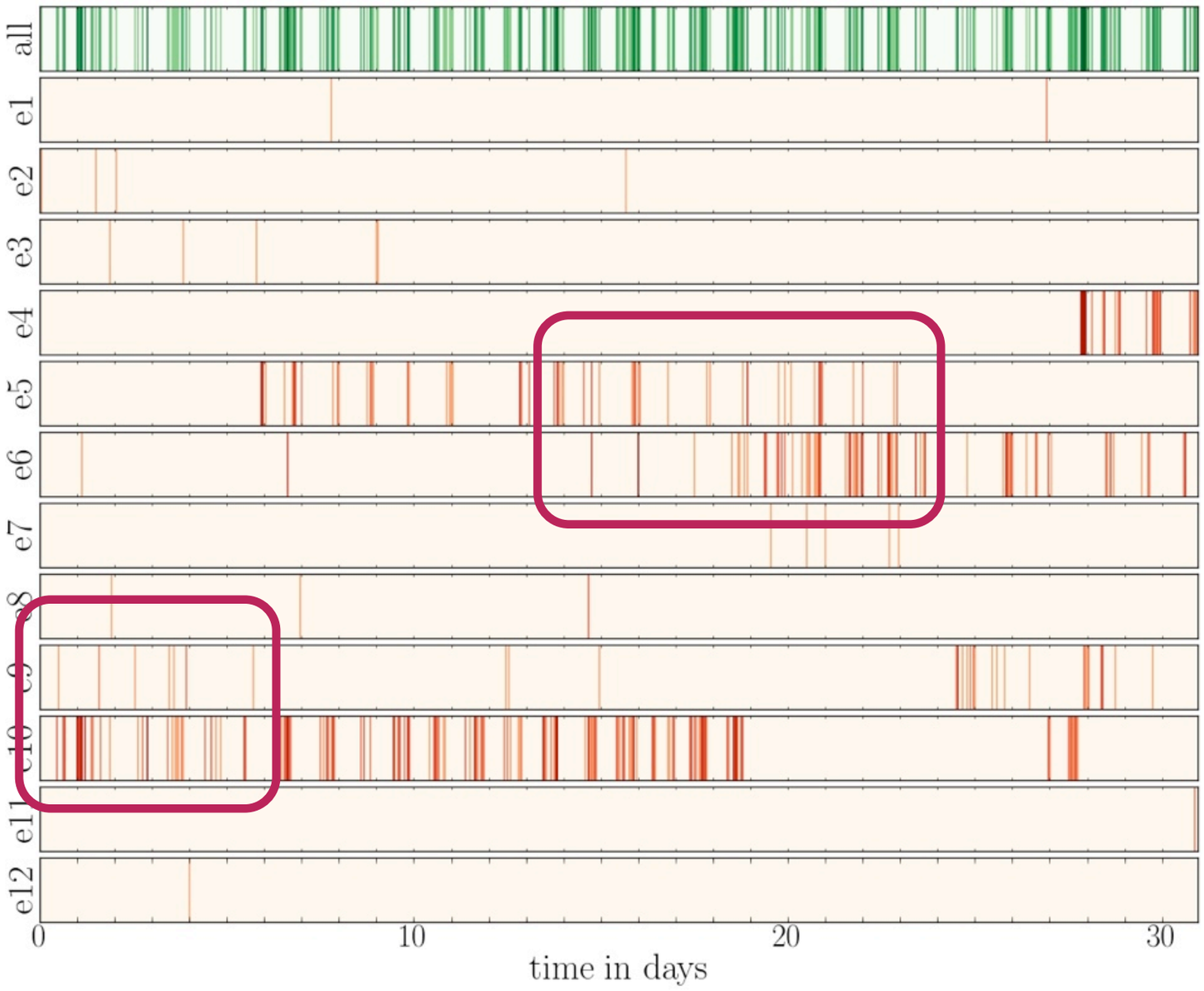
(b) Short message sequence

(c) Email sequence

$$P(E) \propto E^{-\gamma}$$

Karsai et al, Scientific Reports **2**, 397 (2012)

Correlations between link sequences



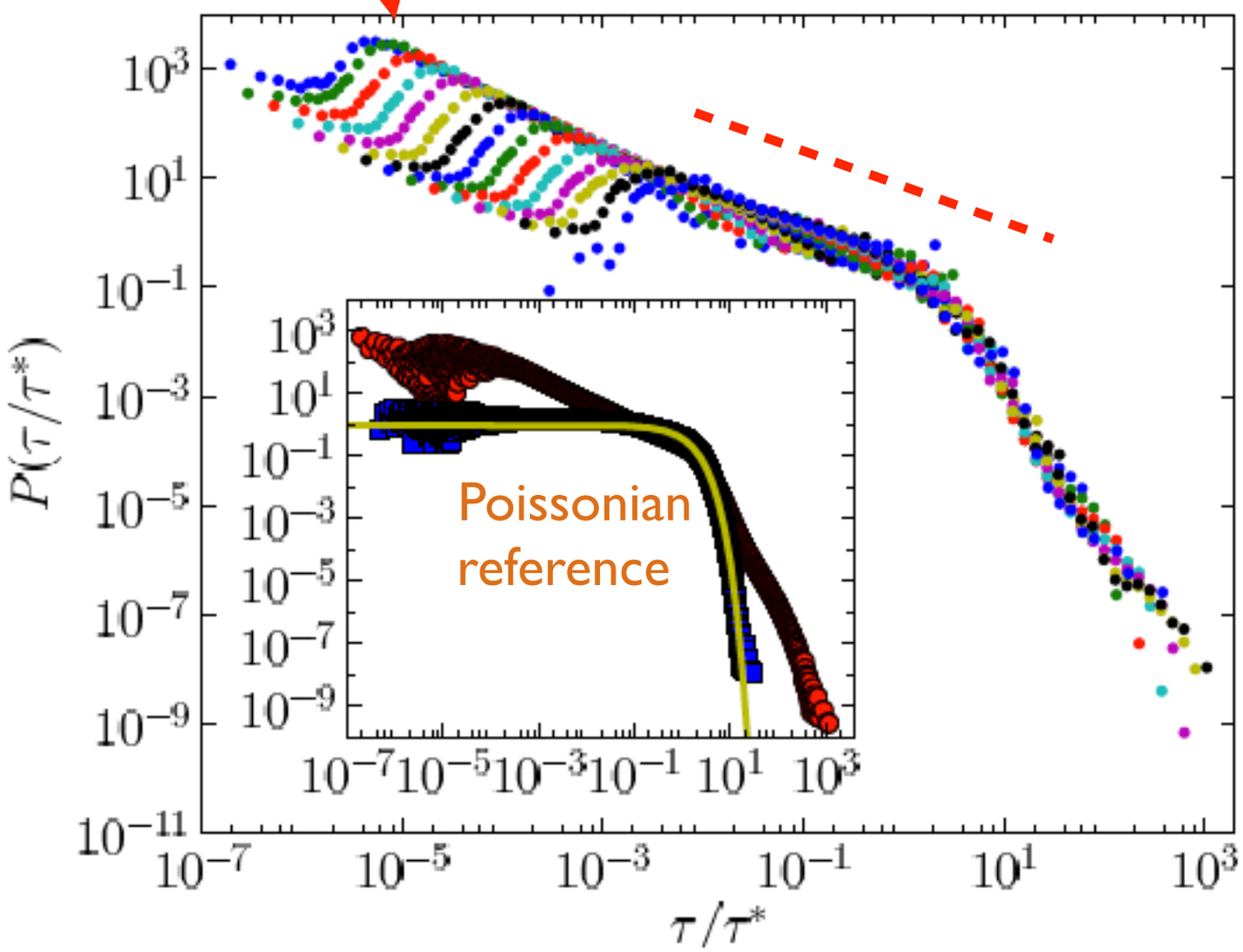
all calls by one person

calls to each acquaintance

Data: time-stamped call records of a mobile operator, see Karsai et al, Phys. Rev. E 83, 025102(R) (2011)

Burstiness: broad inter-event time distributions

this non-scaling part is because of other correlations - we'll get there soon...



- Events = phone calls
- Nodes binned according to # of calls of node
- Inter-event time distribution scaled by average inter-event time in bin

Triggered correlation functions

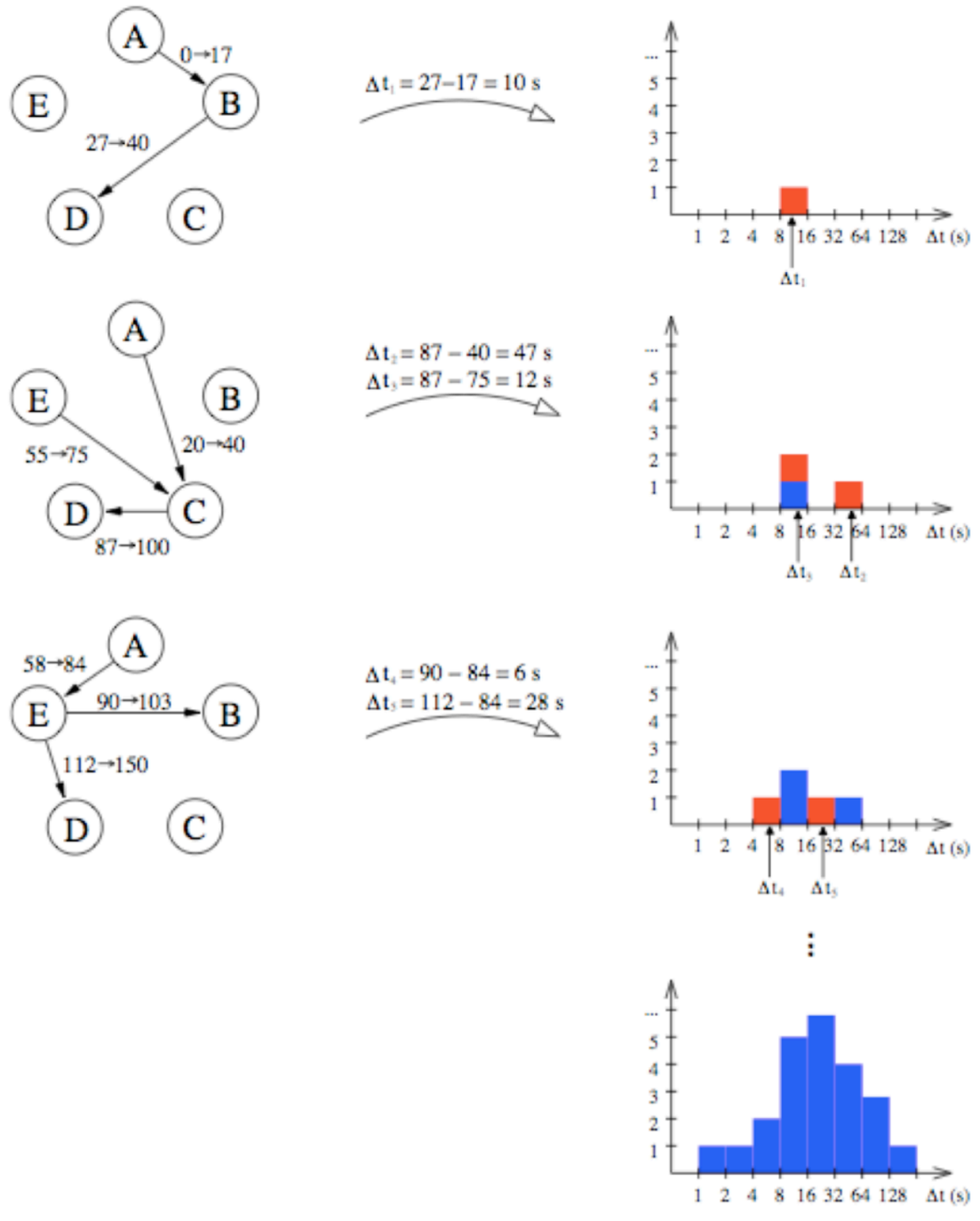
Idea: when a link of node i has an event, calculate Δt to previous events on its links

Plot PDF of Δt

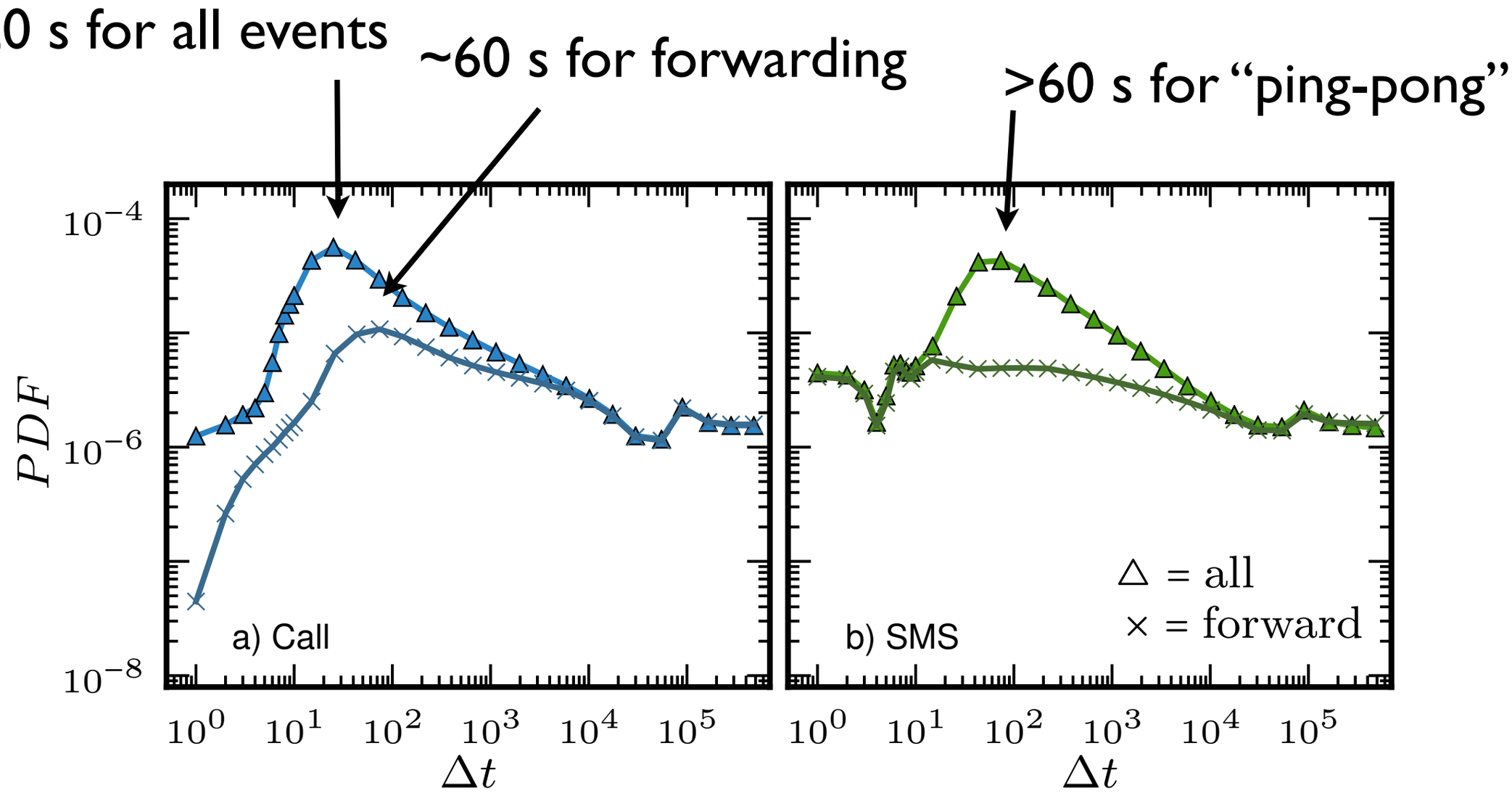
Variants:

a) only calculate Δt for incoming events when there is an outgoing event

b) only calculate events on different links than the trigger



Triggered correlation functions



calls

text messages

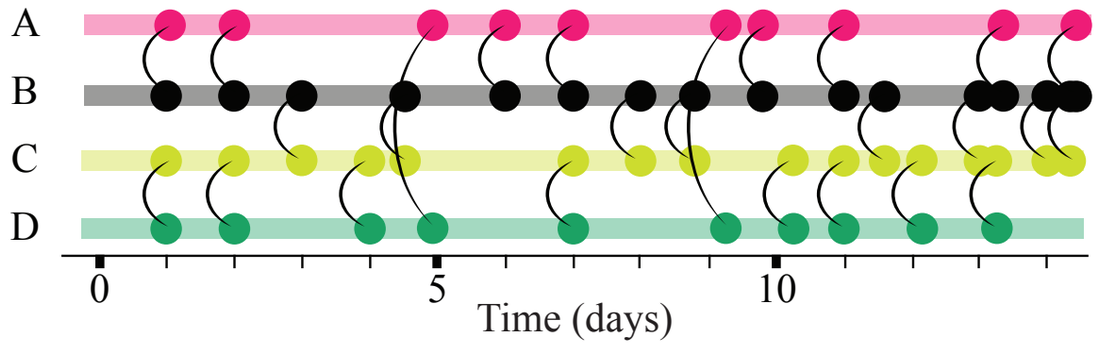
Backlund et al, arXiv:1403.1177 (2014)

Summary

- Human communication: burstiness
- Correlations within burstiness
- Correlations between adjacent links
- Network-level patterns

Temporal Networks

4. Temporal motifs



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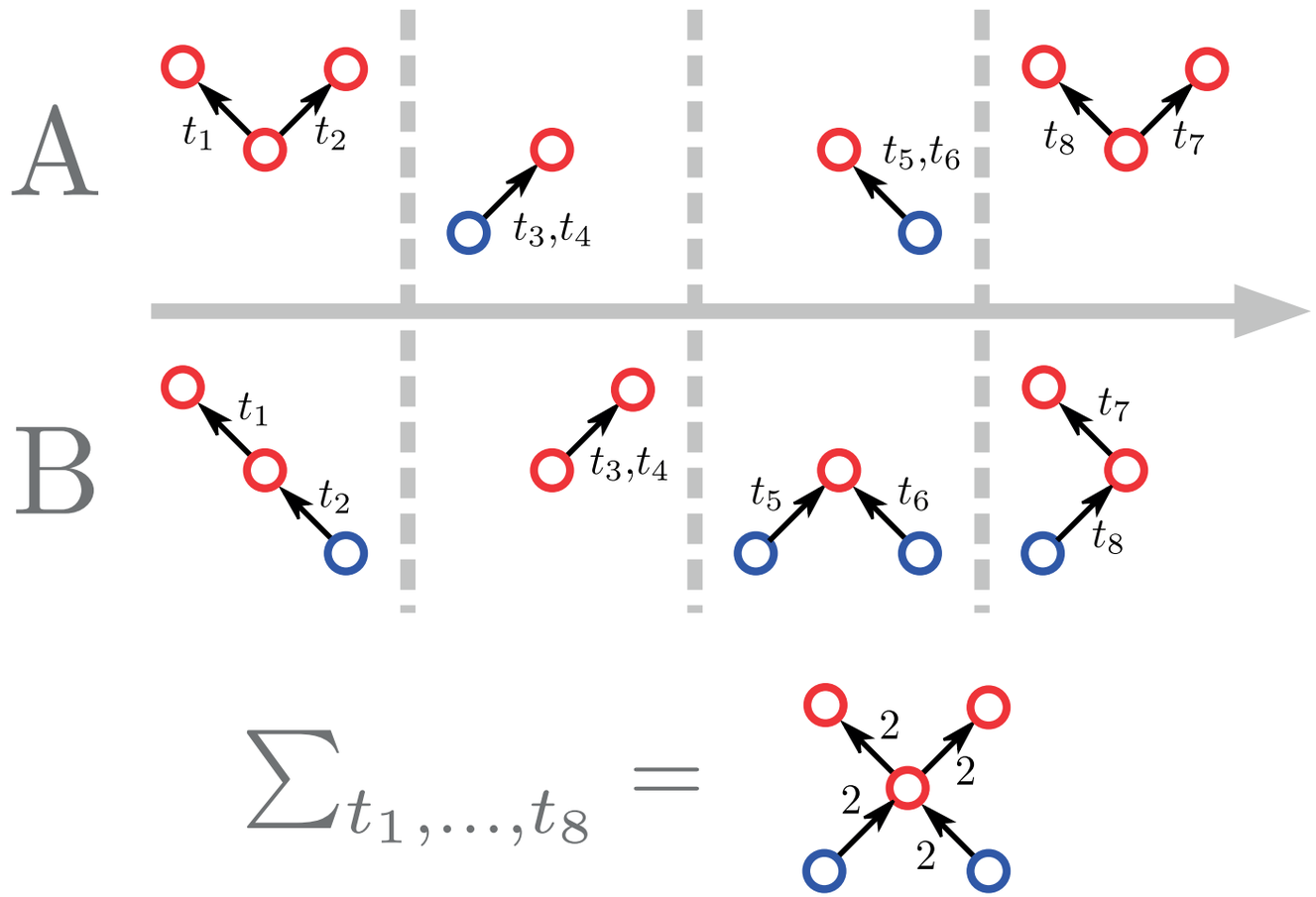
Temporal motifs

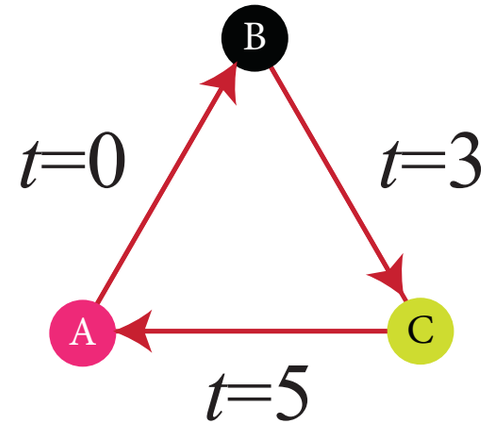


$$\sum_{t_1,\ldots,t_8} =$$

...these two different event sequences look the same when aggregated.

Temporal motifs

- We want to detect **temporal patterns, where links activate within short time periods**



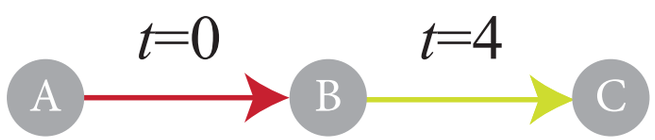
- Patterns should be grouped into equivalence classes (motifs) that take event order into account

- Starting point: define **temporal subgraphs**

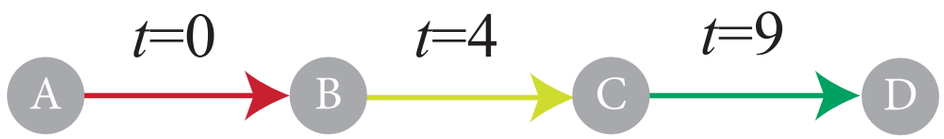
Δt-adjacent and Δt-connected events

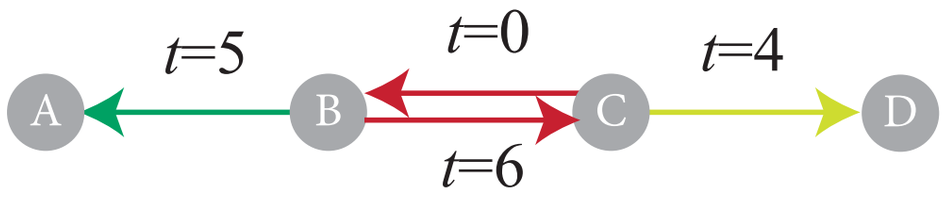
Δt-adjacent: events share a node; follow within Δt

Δt-connected: by Δt-adjacency



events Δt-adjacent for $\Delta t=5$



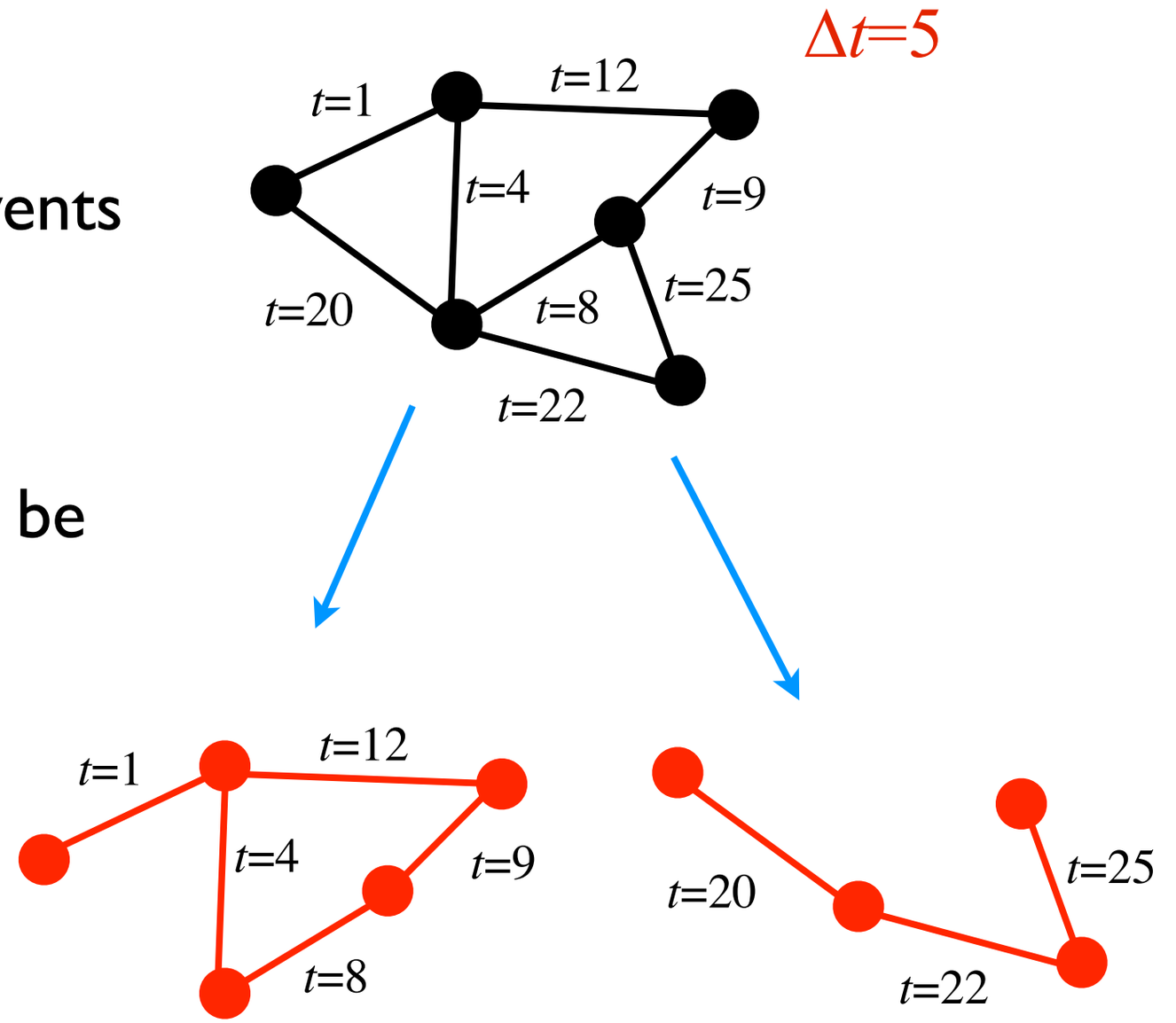


events Δt-connected for $\Delta t=5$

See Kovanen et al., J Stat Mech P11005 (2011)

Temporal subgraphs

- **Temporal subgraph**
  = set of Δt-connected events
- **Maximal temporal subgraph** = no more Δt-connected events can be added

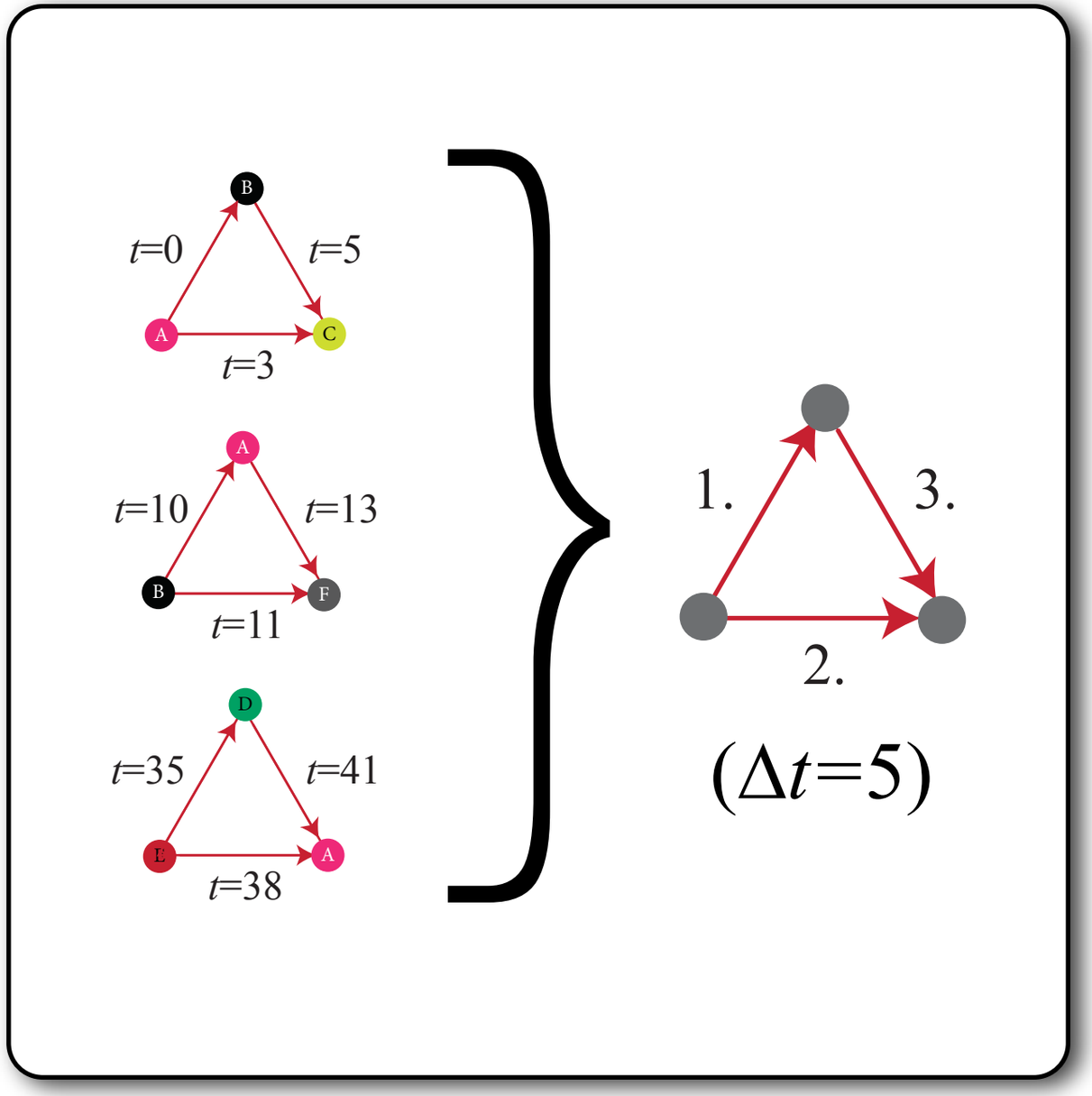


Temporal motifs: definition

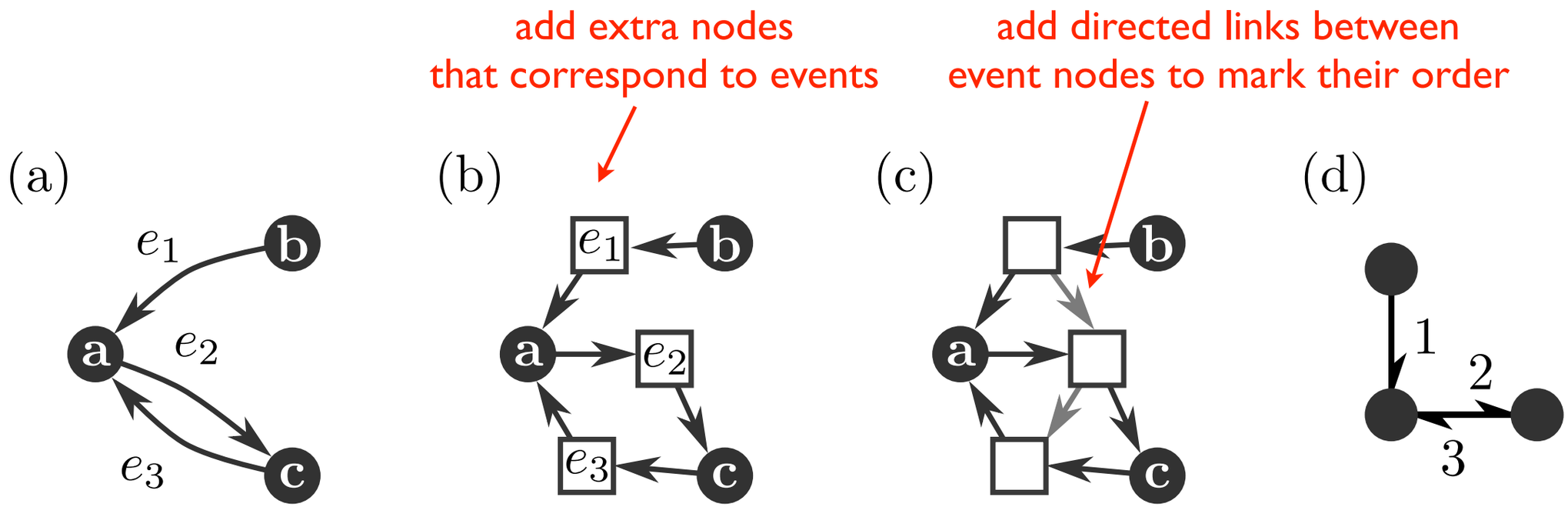
- **temporal motifs** = **equivalence classes** of **valid** temporal subgraphs

- **equivalence**: order of events matters, exact timings don't

- **valid** = no events are skipped at nodes



From subgraphs to equivalence classes



- Temporal subgraphs are mapped into **directed, coloured graphs**
- Isomorphism algorithms are used for assigning equivalence classes

See Kovanen et al., J Stat Mech P11005 (2011)

1

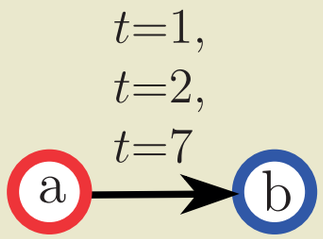
2

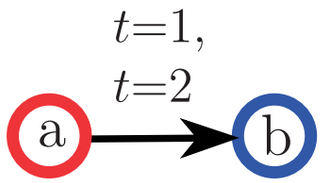
3

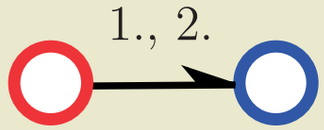
event sequence

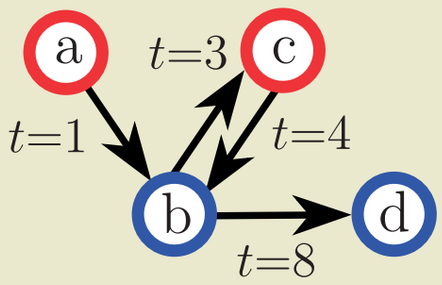
temporal subgraph (Δt=3)

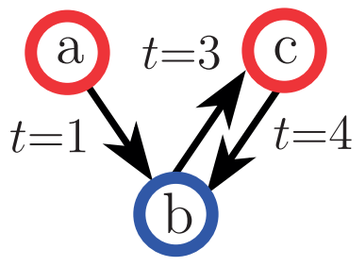
temporal motif

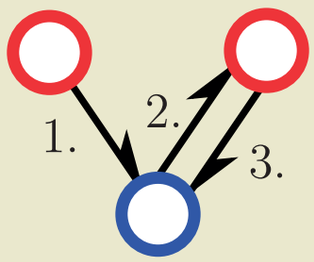






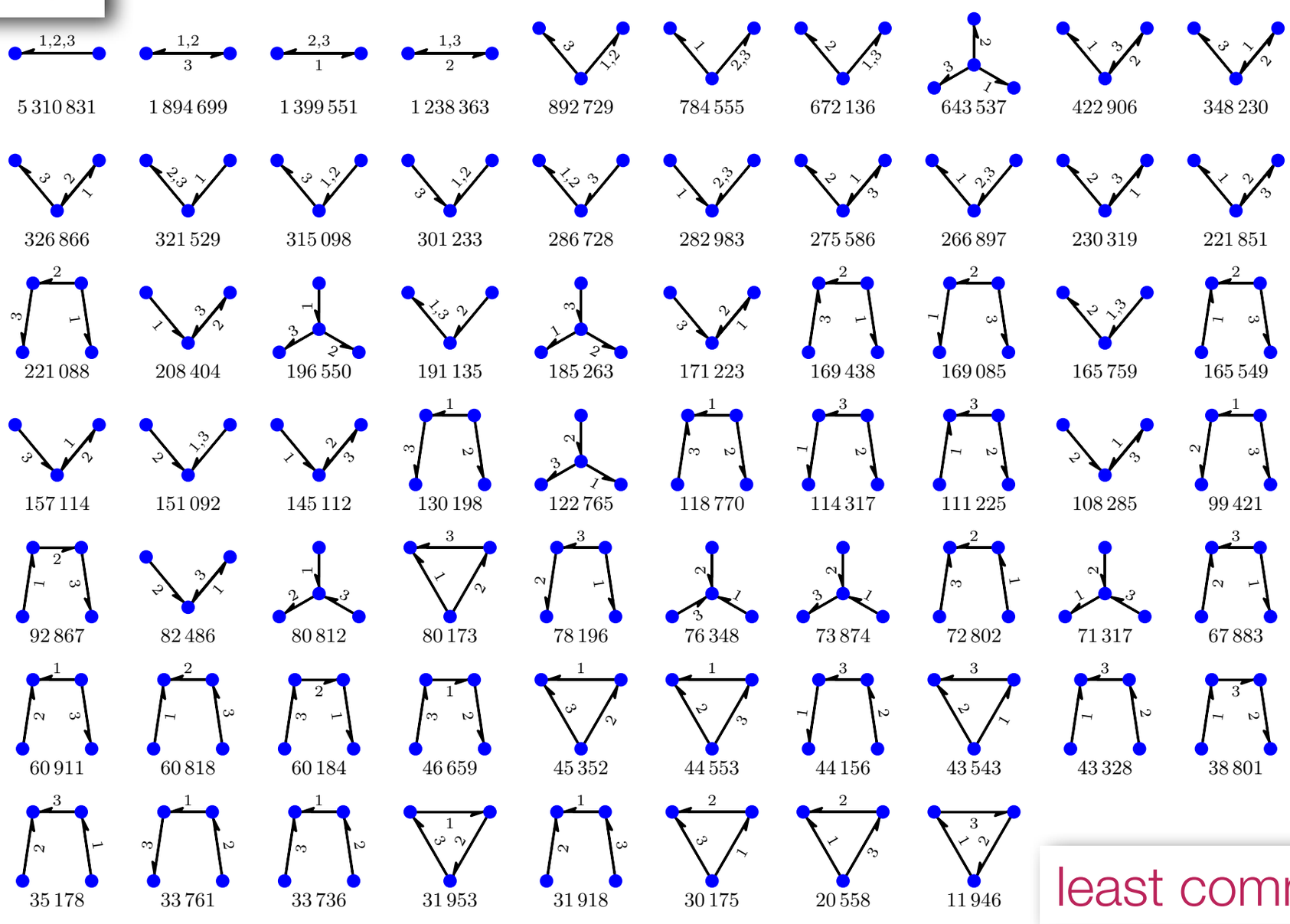






3-event motifs in call data

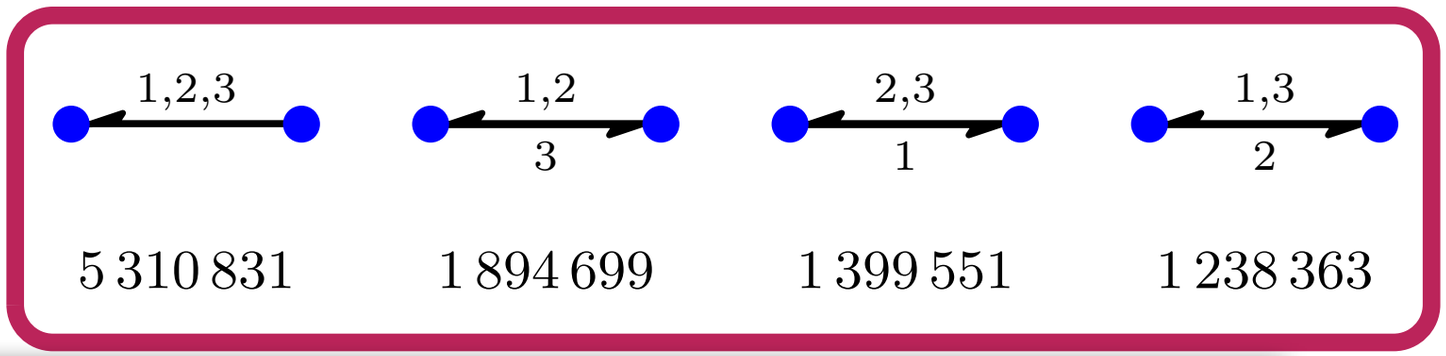
most common



least common

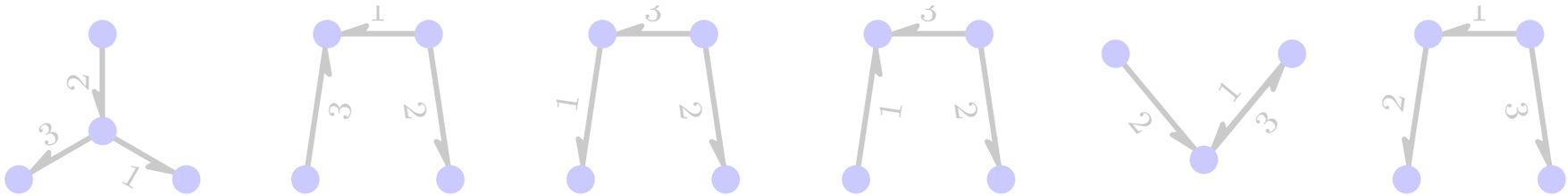
(motifs from ~320 mill. call events, with Δt=10 min → 35% of events are time-adjacent with some other event)

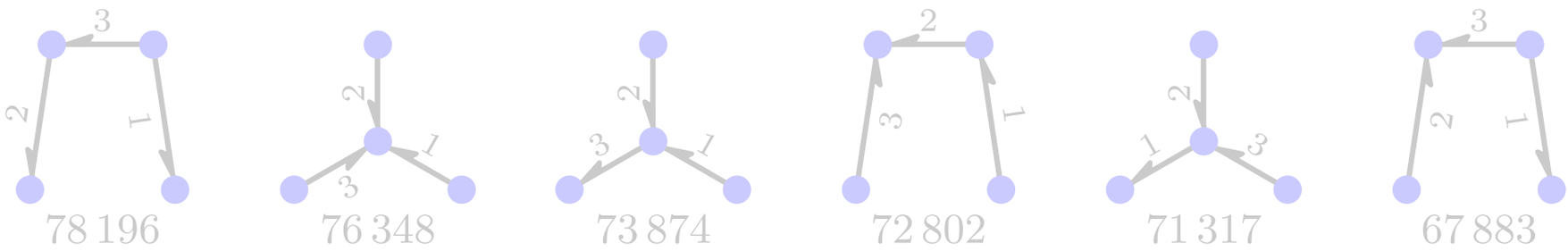
Motifs in call data

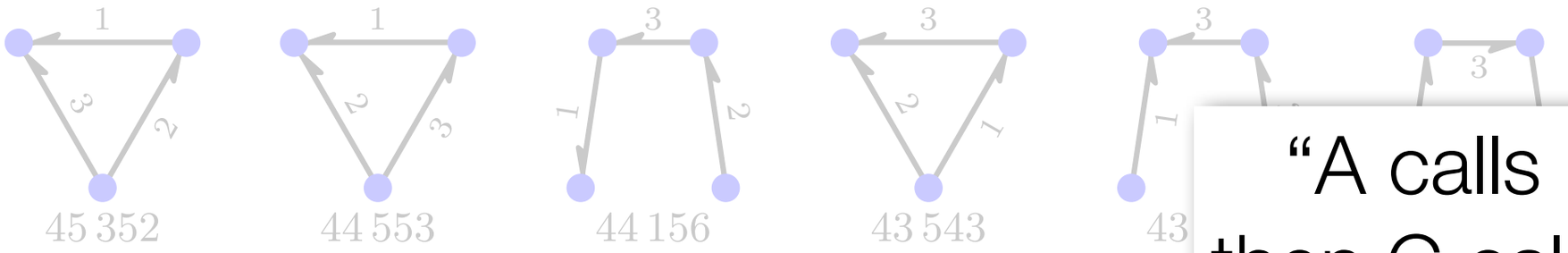


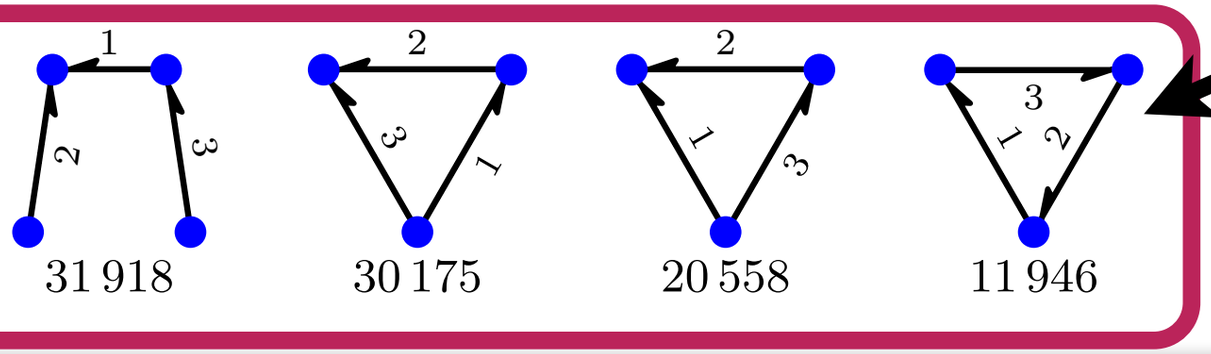
THE MOST COMMON MOTIFS REFLECT LINK BURSTINESS

Motifs in call data









"A calls B, then C calls A, then B calls C"

THE LEAST COMMON MOTIFS ARE (probably) NOT CAUSAL SEQUENCES

Temporal motifs & node attributes

- Assign classes to nodes based on attributes

- E.g. call data: gender, age group, their combination...

- These can be directly used as **node colors** for isomorphism tests

- Reference model:

  "Temporal motif counts do not depend on node attributes, given the weighted aggregated network"

- Calculate z-score

Temporal motifs & node attributes

- Reference model:

  "Temporal motif counts do not depend on node attributes, given the weighted aggregated network"

- The z-score

real count → $C(m)$; mean in ref → $\mu(\widetilde{C}(m))$; std in ref → $\sigma(\widetilde{C}(m))$

$$z(m) = \frac{C(m) - \mu(\widetilde{C}(m))}{\sigma(\widetilde{C}(m))}$$

tells if the null hypothesis is true or not

Temporal motifs & node attributes

- Reference model:

  "Temporal motif counts do not depend on node attributes, given the weighted aggregated network"

- The ratio r

real count → $C(m)$; mean in ref → $\mu(\widetilde{C}(m))$

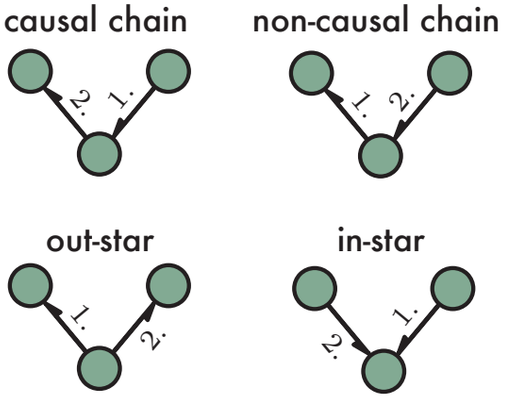
$$r(m) = C(m)/\mu(\widetilde{C}(m))$$

indicates effect size

temporal motifs in call sequences

- there is ***temporal homophily*:**
  patterns where participants are similar (age, gender) are overexpressed

- there are ***gender differences*:**



	Female	Male
Repeated calls	1.11, 1.11	1.13, 1.10
Returned calls	1.02, 1.01	1.02, 1.02
N-Chain	**1.08**, **1.02**	1.01, 1.04
C-Chain	**1.08**, **1.01**	**0.98**, **1.03**
Out-star	**1.10**, **1.03**	**1.01**, **1.04**
In-star	**1.11**, **1.03**	*1.01*, *1.05*

r-value for all-female motifs

r-value for the rest

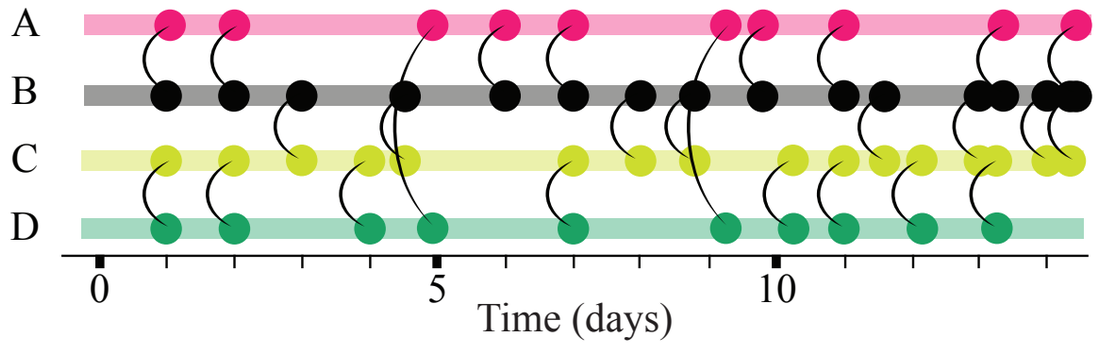
females: more chains & stars

- there are ***group talk patterns:***
  chains & stars overexpressed within cliques

Kovanen, Kaski, Kertész, Saramäki, PNAS 2013

Temporal Networks

5. Temporal network models



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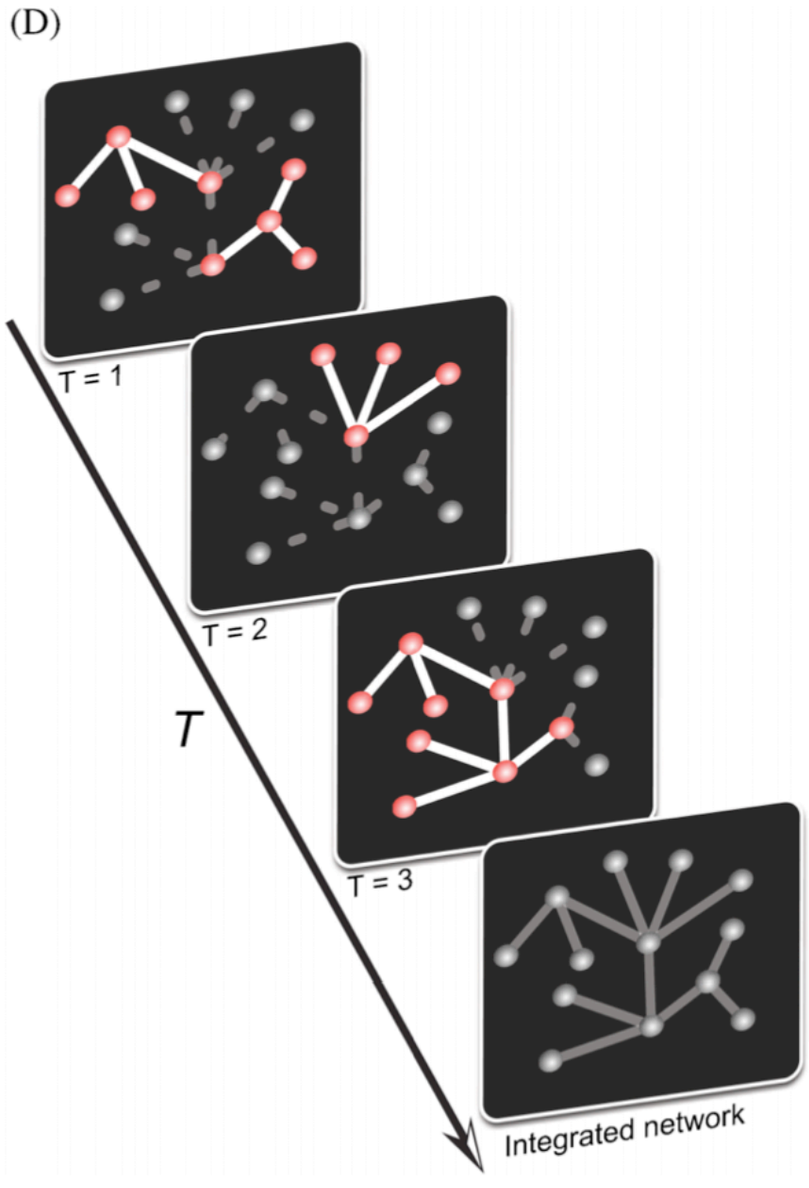
Temporal network models

- There is no "small-world" or "Barabási-Albert" equivalent
- More degrees of freedom = more choices to make = different types of models

Activity-driven model

- By Perra et al (Sci Rep 2, 469, 2012)
- Each node i has an activity level a_i drawn from some distribution $P(a)$
- At each time step t:
  - Begin with N disconnected nodes
  - Each node becomes active with probability $a_i \Delta t$
  - Active nodes make m undirected links to random nodes
  - Links active for Δt, then disappear.



Activity-driven model

- Whole system entirely determined by $P(a)$
- No memory, system is Markovian
- Very easy to simulate
- Simple enough for analytical solutions
  - epidemic thresholds
  - efficiency of control strategies
- Non-Markovian version with memory: Karsai et al, Sci Rep. 2014; 4: 4001

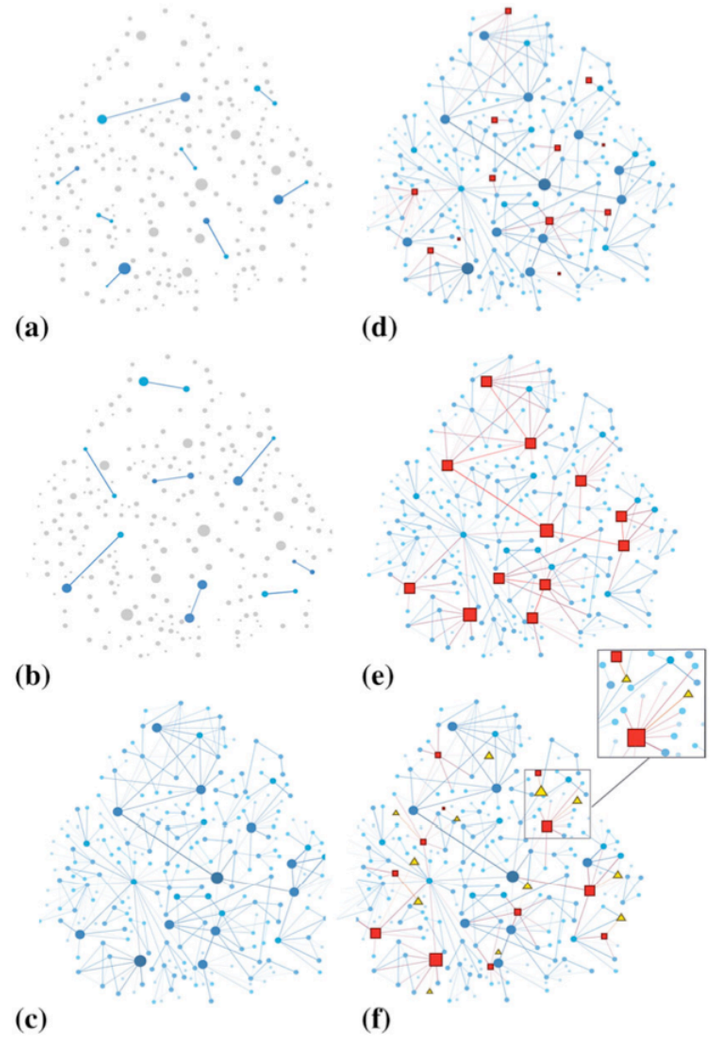
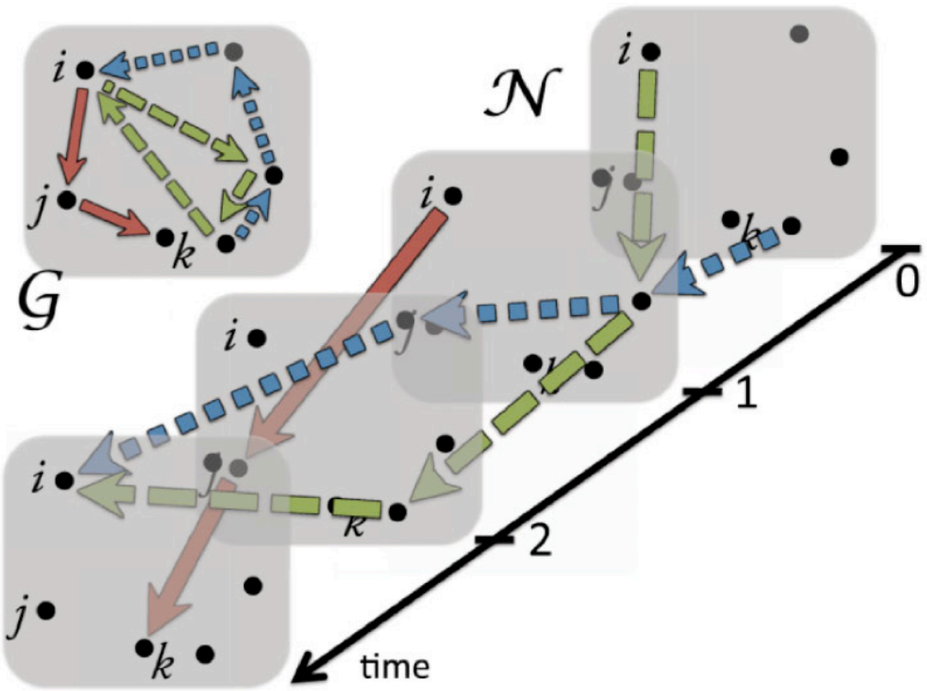


FIG. 1 (color online). Schematic representation of activity driven model and control strategies. (a),(b) Temporal network at two different time steps T_1 and T_2. (c) Integrated network over a certain period of time. The size and color of each node describes its activity, while the width and color of each link describes the weight. (d)–(f) Random, targeted, and egocentric control strategy respectively. Immunized nodes are plotted as red squares; probes as yellow triangles.

Liu et al, PRL 112, 118702 (2014)

Random itinerary model

- Barrat et al, PRL 110, 158702 (2013)
- Target: make a network temporal by assuming static network = superposition of random itineraries



- Input: weighted directed graph, W
  - First draw properties of random walks: 1) starting time, 2) starting node, 3) length, 4) residence time at each node
  - Generate random walks (= time-respecting paths); when walk crosses edge, subtract 1 from weight
  - Links of weight 0 no longer used for walks

Random itinerary model

- Yields temporal networks whose properties when aggregated match with original
- Burstiness of events (from residence time distribution)
- Correlations, frequent paths

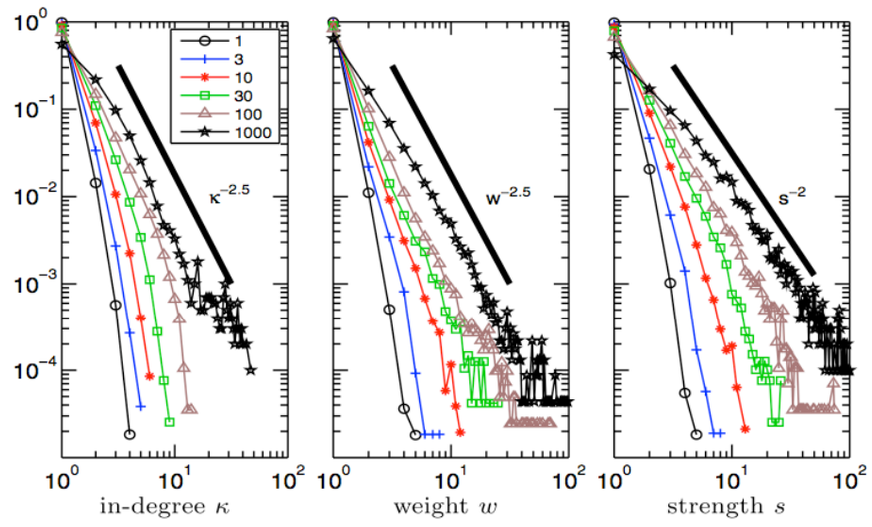
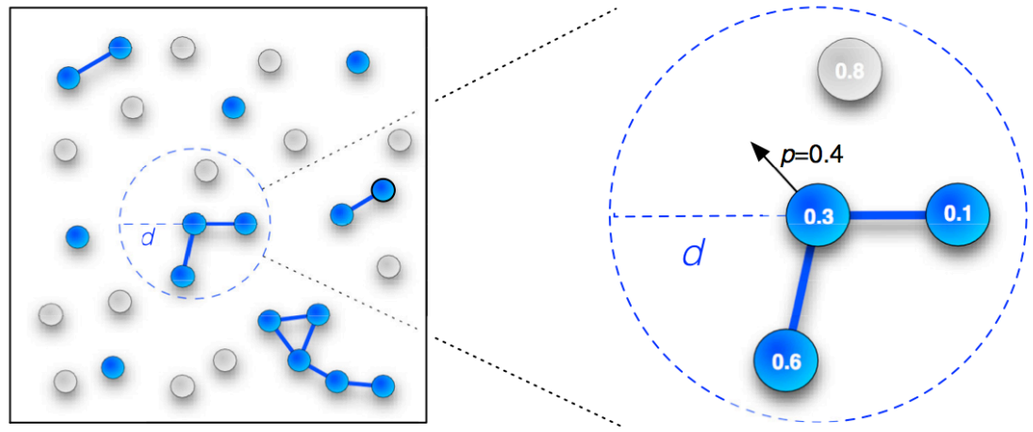


FIG. 3 (color online). Distributions of node in degrees, link weights, and node strengths for networks aggregated over intervals of various lengths. For integration lengths $<10^3$, the figures display distributions obtained by averaging over all corresponding intervals in $\mathcal{W}$. Power laws (thick black lines) with expected exponents are plotted for reference.

Barrat et al, PRL 110, 158702 (2013)

Model of face-to-face interactions

- Starnini et al, PRL 110, 168701 (2013)
- Agents on 2D lattice with periodic BC's
- Each agent endowed with *attractiveness* $a_i \in [0,1)$, from distribution $P(a)$
- Agents walk randomly (random direction/length) or stay put
- Probability of staying put = $\max_{j \in \mathcal{N}_i(t)} \{a_j\}$,
- Neighbours = agents within radius d



- Isolated agents may become inactive with $p=r_i$; inactive agents may activate with $p=1-r_i$; r_i from uniform distribution

Model of face-to-face interactions

- Captures well the properties of real-world data on F2F interactions:
  - contact duration and inter-contact time distributions
  - sublinearly growing degree

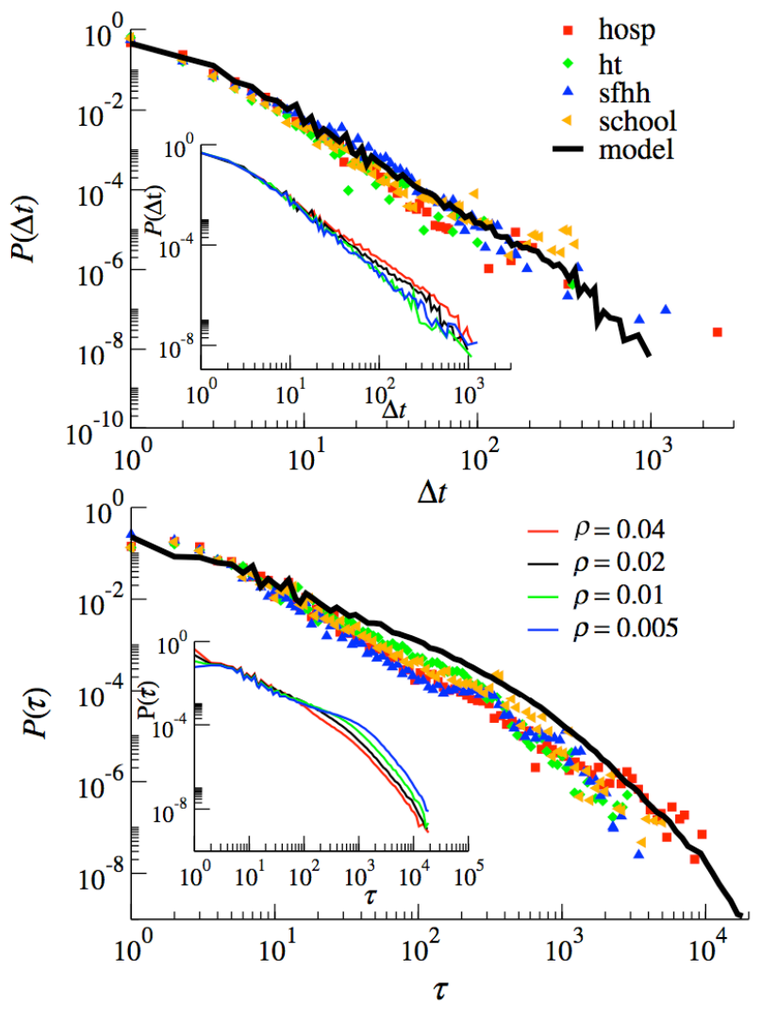


FIG. 2 (color online). Distribution of the contact duration, $P(\Delta t)$, (top) and distribution of the time interval between consecutive contacts, $P(\tau)$, (bottom) for various data sets and for the attractiveness model. Insets: Same distributions for the attractiveness model with different density. Symbols refer to empirical data; lines to results of the model, for different densities ρ.

Future outlook - what's missing?
A somewhat random list.

1. **Temporal communities?**
2. Effects of **higher-order temporal structures**
   - beyond burstiness?
3. **Recurrent temporal structures** (not just patterns, but patterns + participating nodes!)
4. **Node (link) roles** inferred from patterns
5. Dealing with **finite observation interval**

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Temporal networks

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ABSTRACT

A great variety of systems in nature, society and technology – from the web of sexual contacts to the Internet, from the nervous system to power grids – can be modeled as graphs of vertices coupled by edges. The network structure, describing how the graph is wired, helps us understand, predict and optimize the behavior of dynamical systems. In many cases, however, the edges are not continuously active. As an example, in networks of communication via e-mail, text messages, or phone calls, edges represent sequences of instantaneous or practically instantaneous contacts. In some cases, edges are active for non-negligible periods of time: e.g., the proximity patterns of inpatients at hospitals can be represented by a graph where an edge between two individuals is on throughout the time they are at the same ward. Like network topology, the temporal structure of edge activations can affect dynamics of systems interacting through the network, from disease contagion on the network of patients to information diffusion over an e-mail network. In this review, we present the emergent field of temporal networks, and discuss methods for analyzing topological and temporal structure and models for elucidating their relation to the behavior of dynamical systems. In the light of traditional network theory, one can see this framework as moving the information of *when* things happen from the dynamical system on the network, to the network itself. Since fundamental properties, such as the transitivity of edges, do not necessarily hold in temporal networks, many of these methods need to be quite different from those for static networks. The study of temporal networks is very interdisciplinary in nature. Reflecting this, even the object of study has many names—temporal graphs, evolving graphs, time-varying graphs, time-aggregated graphs, time-stamped graphs, dynamic networks, dynamic graphs, dynamical graphs, and so on. This review covers different fields where temporal graphs are considered, but does not attempt to unify related terminology—rather, we want to make papers readable across disciplines.
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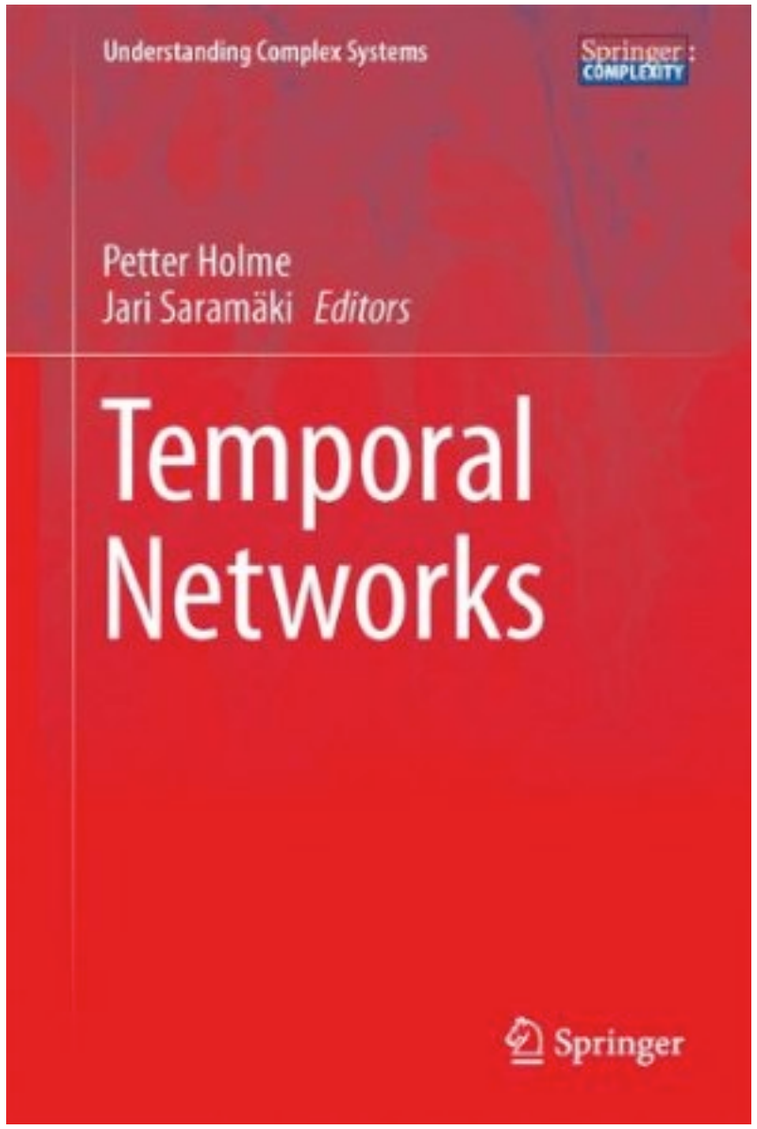
Holme & Saramäki
Physics Reports **519** (2012)



Holme & Saramäki
Springer Complexity Series (2013)